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## Semi-Annual Progress Report on COMPRESSOR AND FAN WAKE CHARACTERISTICS

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B. Reynolds, C. Hah, B. Lakshminarayana, and A. Ravindranath

NASA Grant NsG 3012  
National Aeronautics & Space Administration  
Lewis Research Center

TURBOMACHINERY LABORATORY  
Department of Aerospace Engineering  
The Pennsylvania State University  
University Park, PA 16802

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COMPRESSOR AND FAN WAKE CHARACTERISTICS  
(NASA Grant NsG 3012)

B. Reynolds, C. Hah, B. Lakshminarayana, and A. Ravindranath

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Department of Aerospace Engineering  
The Pennsylvania State University  
University Park, PA 16802

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## PREFACE

The progress on the research carried out under NASA Grant NsG 3012 is reported briefly here. The period covered is for six months ending December 31, 1977.

B. Lakshminarayana  
Principal Investigator

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## 1. STATEMENT OF THE PROBLEM

Both the fluid dynamic properties of a compressor rotor, governing efficiency, and the acoustical properties, governing the radiated noise, are dependent upon the character of the wake behind the rotor. There is a mixing of the wake and the free stream and a consequential dissipation of energy, known as mixing loss. The operational rotors generate the radial component of velocity as well as the tangential and axial components and operate in a centrifugal force field. Hence, a three-dimensional treatment is needed.

The objective of this research is to develop an analytical model for the expressed purpose of learning how rotor flow and blade parameters and turbulence properties such as energy, velocity correlations, and length scale affect the rotor wake characteristics and its diffusion properties. The model will necessarily include three-dimensional attributes. The approach is to employ, as information for the model, experimental measurements and instantaneous velocities; and turbulence properties at various stations downstream from a rotor. A triaxial probe and a rotating conventional probe, which is mounted on a traverse gear operated by two step motors, will be used for these measurements. The experimental program would include the measurement of mean velocities, turbulence quantities across the wake at various radial locations and downstream stations. The ultimate objective is to provide a rotor wake model, based on theoretical analysis and experimental measurements, which the acousticians could use in predicting the discrete as well as broadband noise generated in a fan rotor. This investigation will be useful to turbomachinery aerodynamists in evaluating the aerodynamic losses, efficiency and optimum spacing between a rotor and stator in turbomachinery.

## 2. THEORETICAL INVESTIGATION

The material presented here is a continuation of the analysis described in reference\* [1].

### 2.1 Analysis Based on k-ε Model

#### 2.1.1 Modification of turbulence dissipation equation to include the effects of rotation and streamline curvature

To describe the turbulence decay, second order dissipation tensor\*\*  $\epsilon_{ij} = \nu(\partial u_i / \partial x_k)(\partial u_j / \partial x_k)$  should be modeled. For simplicity of calculation and modeling, the scalar representation of dissipation is usually employed in representing the turbulent flow field. Scalar dissipation equation can be derived through the contraction of dissipation tensor  $\epsilon = \epsilon_{ii}$ . In stationary generalized coordinate system, the transport equation for  $\epsilon$  can be written as follows for high turbulent Reynolds number flows by retaining only higher order terms [2,3]

$$\begin{aligned} \frac{\partial \epsilon}{\partial t} + U_j^j \epsilon_{,j} = & -\nu [U_{i,j} \overline{(s'_{ik} u_{,j}^j)} + U_{k,j} \overline{(s'_{ik} u_{,j}^j)} + U_{,j}^i \overline{(s'_{ik} u_{,j}^j)} + U_{,j}^k \overline{(s'_{ik} u_{,j}^j)}] \\ & \text{I} \\ & -\nu [s'_{u,k}^{ik} u_{i,j} + s'_{u,i}^{ik} u_{k,j} + s'_{ik}^{u,j,k} u_{,j}^i + s'_{ik}^{u,j,i} u_{,j}^k] \\ & \text{II} \\ & -2\nu g^{nj} g^{li} g^{mk} [\overline{s'_{ik,n} s'_{lm,j}} + \overline{s'_{lm,n} s'_{ik,j}}] - \overline{(\epsilon u^j)_{,j}} \\ & \text{III} \qquad \qquad \qquad \text{IV} \\ & - \frac{2}{\rho} \nu g^{li} g^{mk} [\overline{s'_{lm} p'_{,ik}} + \overline{s'_{ik} p'_{,lm}}] . \end{aligned} \quad (1)$$

Term I of the above equation is lower order for flows with straight streamline but is larger for flows with streamline curvature and hence it is retained.

In a rotating frame, additional terms due to centrifugal and Coriolis forces enter into the momentum, dissipation equations. The additional term which appear in the right hand side of Eq. 1 is as follows [5]

\*Numbers in square brackets denote the references listed at the end of this report.  
 \*\*A list of notations is given in Appendix II.

$$\begin{aligned}
& -2\Omega^p [\overline{\epsilon_{ipq} s^{ik} u_{,k}^q} + \overline{\epsilon_{kpq} s^{ik} u_{,i}^q}] \\
& -2\Omega^p [\overline{\epsilon_{lpq} s^{lm} u_{,m}^q} + \overline{\epsilon_{mpq} s^{lm} u_{,l}^q}]
\end{aligned} \tag{2}$$

The  $\Omega^3$  component in Cartesian frame turns out as follows

$$-4\Omega^3 \left[ -\frac{\overline{\partial u_2}}{\partial x_1} \frac{\overline{\partial u_3}}{\partial x_3} + \frac{\overline{\partial u_1}}{\partial x_2} \frac{\overline{\partial u_3}}{\partial x_3} + \frac{\overline{\partial u_3}}{\partial x_1} \frac{\overline{\partial u_2}}{\partial x_3} - \frac{\overline{\partial u_3}}{\partial x_2} \frac{\overline{\partial u_1}}{\partial x_3} \right] \tag{3}$$

and becomes identically zero for isotropic flow. The effects of rotation comes through the anisotropy of turbulence. In the process of turbulence modeling, if local isotropy of turbulence structure is assumed, the effects of rotation can be neglected. But, in the rotor wake, the turbulence is not isotropic. As proposed by Rotta, the deviation from isotropy can be written as [6]

$$\overline{u_i u_j} - \frac{2}{3} k \delta_{ij} \tag{4}$$

and the effects of rotation can be represented through the following term

$$c_1 \left( \overline{\frac{u_i u_j}{2k}} - \frac{1}{3} \delta_{ij} \right) \epsilon \Omega^p \tag{5}$$

where  $c_1$  is universal constant. If we introduce Rossby number ( $U/L\Omega$ ) in Eq. 5 it can be also represented as follows

$$c_1 \left( \overline{\frac{u_i u_j}{2k}} - \frac{1}{3} \delta_{ij} \right) (\epsilon^2/k) (\text{Rossby number})^{-1} \tag{6}$$

The modeling of other terms in dissipation equation in rotating coordinate system can be successfully carried out much the same way as in stationary coordinate system. If we adopt the modeling of Eq. 1 by the Imperial College group [7,8], the dissipation equation in rotating coordinate system can be shown to be as follows

$$\begin{aligned}
\frac{D\epsilon}{Dt} = & -c_{\epsilon 1} \frac{\overline{\epsilon u_i u_k}}{k} \frac{\partial U_i}{\partial x_k} - c_{\epsilon 2} \frac{\epsilon^2}{k} - c_{\epsilon} \frac{\partial}{\partial x_k} \left( \overline{\frac{k}{\epsilon} u_k u_l} \frac{\partial \epsilon}{\partial x_l} \right) \\
& + c_1 \left( \overline{\frac{u_i u_k}{2k}} - \frac{1}{3} \delta_{ij} \right) \left( \frac{\epsilon^2}{k} \right) (\text{Rossby number})^{-1}
\end{aligned} \tag{7}$$

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The effects of streamline curvature in the flow are included implicitly in the momentum conservation equation. If the turbulence modeling is performed with sufficient lower order terms, the effects of streamline curvature can be included in the modeled equation. But for the simplicity of calculation, the turbulence modeling usually includes only higher order terms. Therefore further considerations of the effects of streamline curvature is necessary. In the modeling process, the turbulence energy equation is treated exactly with the exception of the diffusion term. The turbulence dissipation equation is thus the logical place to include the model for the effects of streamline curvature.

Bradshaw [4] has deduced that it is necessary to multiply the secondary strain associated with streamline curvature by a factor of ten for the accounting of the observed behavior with an "effective viscosity" transport model. In the process of modeling of turbulence dissipation equation, the lower order generation term has been retained for the same reason. In our modeling the effects of streamline curvature are included in the generation term of turbulence of dissipation equation through Richardson number defined as follows,

$$R_i = \left[ \frac{2v_\theta \cos\alpha}{r^2} \frac{\partial}{\partial y}(rv_\theta) \right] / \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( r \frac{\partial v_\theta / r}{\partial y} \right)^2 \right] \quad (8)$$

Hence the final turbulence dissipation equation which include both the rotation and streamline curvature effects is given by

$$\frac{D\varepsilon}{Dt} = -c_{\varepsilon_1} (1 + C_{R_i}) \frac{\overline{\varepsilon u_i u_k}}{k} \frac{\partial U_i}{\partial x_k} - c_{\varepsilon_2} \frac{\varepsilon^2}{k} + c_\varepsilon \frac{\partial}{\partial x_k} \left( \frac{k}{\varepsilon} \overline{u_k u_\ell} \frac{\partial \varepsilon}{\partial x_\ell} \right) + c_1 \left( \frac{\overline{u_i u_k}}{2k} - \frac{1}{3} \delta_{ij} \right) \left( \frac{\varepsilon^2}{k} \right) \quad (9)$$

x(Rossby number)

where  $c_{\varepsilon_1}$ ,  $c_c$ ,  $c_{\varepsilon_2}$ ,  $c_\varepsilon$ , and  $c_1$  are universal constants.

### 2.1.2 Turbulence closure

The Reynolds stress term which appear in the time mean turbulent momentum equation should be rationally related to mean strain or another differential equation for Reynolds stresses should be introduced to get closure of the governing

equations. For the present study, k-ε model which was first proposed by Harlow and Nakayama [9] is applied and further developed for the effects of rotation and streamline curvature. In k-ε model, the Reynolds stresses are calculated via the effective eddy viscosity concept and can be written as follows:

$$\overline{-u_i u_j} = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2k}{3} \delta_{ij} \quad (10)$$

The second term in the right hand side makes the average normal components of Reynolds stresses zero and turbulence pressure is included in the static pressure. Effective eddy viscosity  $\nu_t$  is determined by the local value of k & ε,

$$\nu_t = c_\mu \frac{k^2}{\epsilon} \quad (11)$$

where  $c_\mu$  is an empirical constant. The above equation implies that the k-ε model characterizes the local state of turbulence with two parameters k and ε. ε is defined as follows with the assumption of homogeneity of turbulence structure.

$$\epsilon = \nu \frac{\partial u_i}{\partial x_k} \frac{\partial u_i}{\partial x_k} \quad (12)$$

For the closure, two differential equations for the distribution of k and ε are necessary. The semi-empirical transport equation of k is as follows

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_k} \left[ \frac{\nu_t}{G_k} \frac{\partial k}{\partial x_k} \right] + G - \epsilon \quad \text{ORIGINAL PAGE IS OF POOR QUALITY} \quad (13)$$

Where G is the production of turbulence energy and  $G_k$  is empirical constant.

In Section 2.1.1, the transport equation of ε has been developed for the effects of rotation and streamline curvature.

### 2.1.3 The numerical analysis of turbulent rotor wakes in compressors

The rotor wakes in a compressor are turbulent and develops in the presence of large curvature and constraints due to hub and tip walls as well as adjoining wakes. Rotating coordinate system should be used for the analysis of rotor wakes because the flow can be considered steady in this system. The above

mentioned three-dimensionality as well as complexity associated with the geometry and the coordinate system make the analysis of rotor wakes very difficult. A scheme for numerical analysis and subsequent turbulence closure is described below.

The equations governing three-dimensional turbulent flow are elliptic and large computer storage and calculation is generally needed for the solution. The time dependent method is theoretically complete, but if we consider the present computer technology, the method is not a practical choice. The concept of parabolic-elliptic Navier-Stokes equation has been successfully applied to some simple three dimensional flows. The partially parabolic and partially elliptic nature of equations renders the possibility of marching in downstream, which means the problem can be handled much like two-dimensional flow. The above concept can be applied to the flow problems which have one dominant flow direction. Jets and wakes flows have one dominant flow direction and can be successfully solved with the concept of parabolic elliptic Navier-Stokes equation. The numerical scheme of rotor wakes with the concept of parabolic elliptic Navier-Stokes equation are described in Appendix II. Alternating direction implicit method is applied to remove the restrictions on the mesh size in marching direction without losing convergence of solution. The alternating-direction implicit method applied in this study is the outgrowth of the method of Peaceman and Rachford [10].

Like cascade problems, the periodic boundary condition arises in the rotor wake problem. It is necessary to solve only one wake region extending from the mid passage of one blade to the mid passage of other blade because of the periodicity in the flow. This periodic boundary condition is successfully treated in alternating direction implicit method in y-implicit step (Appendix II]. It is desirable to adopt the simplest coordinate system in which the momentum and energy equations are expressed. As mentioned earlier the special coordinate system should be rotating with rotor blade.

The choice of special frame of the reference depends on the shape of the boundary surface and the convenience of calculation. But for the present study, the marching direction of parabolic elliptic equation should be one coordinate direction. The boundary surfaces of present problem are composed of two concentric cylindrical surfaces (if we assume constant annulus area for the compressor) and two planar periodic boundary surfaces. One apparent choice is cylindrical polar coordinate system, but it can not be adopted because marching direction does not coincide with one of the coordinate axis. No simple orthogonal coordinate system can provide a simple representation of the boundary surfaces (which means boundary surfaces coincide with the coordinate surface, for example,  $x_i = \text{constant}$ ). In view of this, the best choice is Cartesian because it does not include any terms arising from the curvature of the surfaces of reference. The overall description is included in Appendix II in detail.

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## 2.2 Modified Momentum Integral Technique

This analysis\* is a continuation and slight modification to that given in Reference 11.

In Reference 12, the static pressure was assumed to be constant across the rotor wake. From measurements taken in the wake of heavily and lightly loaded rotors a static pressure variation was found to exist across the rotor wake. These results indicate that the assumption of constant static pressure across the rotor wake is not valid. The following analysis modifies that given in Reference 11 to include static pressure variation.

Also included in this analysis are the effects of turbulence, blade loading, and viscosity on the rotor wake. Each of these effects are important in controlling the characteristics of the rotor wake.

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\*Even though Equations 14 through 43 given in this report are a slightly modified version of the analysis presented in Reference 11, the entire analysis is repeated for completeness. The analysis subsequent to Equation 43 is new.

The equations of motion in cylindrical coordinates are given below for  $r$ ,  $\theta$ , and  $z$ , respectively (see Figure 1 for the coordinate system)

$$\frac{v}{r} \frac{\partial U}{\partial \theta} + U \frac{\partial U}{\partial r} + W \frac{\partial U}{\partial z} - \frac{1}{r} (V + \Omega r)^2 = - \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{\partial \overline{u'^2}}{\partial r} - \frac{\overline{u'^2}}{r} + \frac{\overline{v'^2}}{r} - \frac{\partial \overline{(u'v')}}{r \partial \theta} - \frac{\partial \overline{(u'w')}}{\partial z} \quad (14)$$

$$\frac{v}{r} \frac{\partial V}{\partial \theta} + U \frac{\partial V}{\partial r} + W \frac{\partial V}{\partial z} + \frac{UV}{r} + 2\Omega U = - \frac{1}{\rho r} \frac{\partial P}{\partial \theta} - \frac{1}{r} \frac{\partial \overline{(v'^2)}}{\partial \theta} - \frac{2 \overline{(u'v')}}{r} - \frac{\partial \overline{(u'v')}}{\partial r} - \frac{\partial \overline{(v'w')}}{\partial z} \quad (15)$$

$$\frac{v}{r} \frac{\partial W}{\partial \theta} + U \frac{\partial W}{\partial r} + W \frac{\partial W}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\partial \overline{(w'^2)}}{\partial z} - \frac{\partial \overline{(u'w')}}{\partial r} - \frac{\overline{(u'w')}}{r} - \frac{1}{r} \frac{\partial \overline{(v'w')}}{\partial \theta} \quad (16)$$

and continuity,

$$\frac{\partial V}{r \partial \theta} + \frac{\partial W}{\partial z} + \frac{\partial U}{\partial r} + \frac{U}{r} = 0 \quad (17)$$

### 2.2.1 Assumptions and simplification of the equations of motion

The flow is assumed to be steady in the relative frame of reference and to be turbulent with negligible laminar stresses.

To integrate Equations 14-17 across the wake, the following assumptions are made.

1. The velocities  $V_0$  and  $W_0$  are large compared to the velocity defects  $u$  and  $w$ . The wake edge radial velocity,  $U_0$ , is zero (no radial flows exist outside the wake). In equation form this assumption is,

$$U = u(\theta, z) \quad (U_0 = 0) \quad (18)$$

$$V = V_0(r) - v(\theta, z) \quad (V_0 \gg v) \quad (19)$$

$$W = W_0(r) - w(\theta, z) \quad (W_0 \gg w) \quad (20)$$

2. The static pressure varies as,

$$p = p(r, \theta, z)$$

3. Similarity exists in a rotor wake.

Substituting Equations 18-20 into Equations 14-17 gives,

$$\begin{aligned} \frac{V_o}{r} \frac{\partial u}{\partial \theta} + W_o \frac{\partial u}{\partial z} - \frac{1}{r}(V_o + \Omega r)^2 - \frac{1}{r}(-2vV_o - 2v\Omega r) = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{\partial \overline{u'^2}}{\partial r} - \frac{\overline{u'^2}}{r} + \frac{\overline{v'^2}}{r} \\ - \frac{\partial \overline{(u'v')}}{r \partial \theta} - \frac{\partial \overline{(u'w')}}{\partial z} \end{aligned} \quad (21)$$

$$\begin{aligned} -\frac{V_o}{r} \frac{\partial v}{\partial \theta} + u \frac{\partial V_o}{\partial r} - W_o \frac{\partial v}{\partial z} + 2\Omega u + \frac{uV_o}{r} = -\frac{1}{\rho} \frac{\partial p}{r \partial \theta} - \frac{1}{r} \frac{\partial \overline{(v'^2)}}{\partial \theta} - \frac{2\overline{(u'v')}}{r} - \frac{\partial \overline{(u'v')}}{\partial r} \\ - \frac{\partial \overline{(v'w')}}{\partial z} \end{aligned} \quad (22)$$

$$-\frac{V_o}{r} \frac{\partial w}{\partial \theta} + u \frac{\partial W_o}{\partial r} - W_o \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - \frac{\partial \overline{(w'^2)}}{\partial z} - \frac{\partial \overline{(u'w')}}{\partial r} - \frac{\overline{(u'w')}}{r} - \frac{1}{r} \frac{\partial \overline{(v'w')}}{\partial \theta} \quad (23)$$

$$-\frac{1}{r} \frac{\partial v}{\partial \theta} - \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \quad (24)$$

### 2.2.2 Momentum integral equations

Integration of inertia terms are given in Reference 12. The integration of the pressure gradient, normal intensity and shear stress terms are given in this section.

The static pressure terms in Equations 21-23 are integrated across the wake in the following manner. For Equation 22,

$$\int_{\theta_c}^{\theta_o} \frac{1}{r} \frac{\partial p}{\partial \theta} d\theta = \frac{1}{r}(p_e - p_c) \quad (25)$$

The integration of the pressure term in Equation 21 is,

$$\int_{\theta_c}^{\theta_o} \frac{\partial p}{\partial r} d\theta = \frac{\partial p}{\partial r} + p_c \frac{\partial \theta_c}{\partial r} - p_e \frac{\partial \theta_o}{\partial r} \quad (26)$$

It is known that,

$$r(\theta_c - \theta_o) = \delta \quad (27)$$

Differentiating the above expression,

$$\frac{\partial}{\partial r}(r\theta_c - r\theta_o) = \frac{\partial \delta}{\partial r} \quad (28)$$

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$$\frac{r \partial \theta_o}{\partial r} = \frac{\partial}{r} - \frac{\partial \delta}{\partial r} + \frac{r \partial \theta_c}{\partial r} \quad (29)$$

Using Equation 29, Equation 26 becomes,

$$\int_{\theta_c}^{\theta_o} \frac{\partial p}{\partial r} d\theta = \frac{\partial \bar{p}}{\partial r} + (p_c - p_e) \frac{\partial \theta_c}{\partial r} - p_e \left( \frac{\delta}{r^2} - \frac{\partial \delta}{r \partial r} \right) \quad (30)$$

Integrating the static pressure term in Equation 23 across the wake.

$$\int_{\theta_c}^{\theta_o} \frac{\partial p}{\partial z} d\theta = \frac{\partial \bar{p}}{\partial z} + \frac{p_c \partial \theta_c}{\partial z} - p_e \frac{\partial \theta_o}{\partial z} \quad (31)$$

where

$$\frac{\partial \theta_c}{\partial z} = \frac{1}{r} \left( \frac{V}{W} \right)_c = \frac{1}{r} \left( \frac{V_o - v_c}{W_o - w_c} \right) \quad (32)$$

and

$$\frac{\partial \delta}{\partial z} = \frac{\partial}{\partial z} (r \theta_c - r \theta_o) \quad (33)$$

$$\frac{\partial \theta_o}{\partial z} = \frac{\partial \theta_c}{\partial z} - \frac{1}{r} \frac{\partial \delta}{\partial z} \quad (34)$$

Substituting Equations 32 and 34 into Equation 31

$$\int_{\theta_c}^{\theta_o} \frac{\partial p}{\partial z} dz = \frac{\partial \bar{p}}{\partial z} + \frac{p_c}{r} \left( \frac{V_o - v_c}{W_o - w_c} \right) - p_e \left[ \frac{1}{r} \left( \frac{V_o - v_c}{W_o - w_c} \right) - \frac{1}{r} \frac{\partial \delta}{\partial z} \right] \quad (35)$$

Integration across the wake of the turbulence intensity terms in Equations 21-23 is given as,

$$\frac{\delta}{r} \int_0^1 \left( - \frac{\partial \overline{u'^2}}{\partial r} - \frac{\overline{u'^2}}{r} + \frac{\overline{v'^2}}{r} \right) d\eta \quad (36)$$

$$\frac{\delta}{r^2} \int_0^1 \left( - \frac{\partial (\overline{v'^2})}{\partial \theta} \right) d\eta \quad (37)$$

$$\frac{\delta}{r} \int_0^1 \left( - \frac{\partial \overline{w'^2}}{\partial z} \right) d\eta \quad (38)$$

Integration of any quantity  $q$  across the wake is given as (Reference 12),

$$\int_{\theta_c}^{\theta_o} \frac{\partial q}{\partial \theta} d\theta = -q_c \quad (39)$$

$$\int_{\theta_c}^{\theta_o} \frac{\partial(q(z, \theta))}{\partial z} d\theta = \frac{d}{dz} \int_{\theta_c}^{\theta_o} q(z, \theta) d\theta + q_c \frac{\partial \theta_c}{\partial z} - q_o \frac{\partial \theta_o}{\partial z} \quad (40)$$

If similarity in the rotor wake exists

$$q = q_c(z)q(\eta) \quad (41)$$

Using Equation 41, Equation 40 may be written as,

$$\int_{\theta_c}^{\theta_o} \frac{\partial(q(z, \theta))}{\partial z} d\theta = \frac{d(q_c \delta)}{rdz} \int_0^1 q(\eta) d\eta + \frac{q_c}{r} \left( \frac{V_o - v_c}{W_o - w_c} \right) \quad (42)$$

From Equation 40, it is also known that,

$$\int_{\theta_c}^{\theta_o} q d\theta = \frac{\delta q_c}{r} \int_0^1 q(\eta) d\eta \quad (43)$$

The shear stress terms in Equation 21 are integrated across the wake as follows. First, it is known that

$$\int_{\theta_c}^{\theta_o} \frac{\partial(\overline{u'w'})}{r\partial\theta} d\theta = \int_{\theta_c}^{\theta_o} \frac{\partial(\overline{u'v'})}{r\partial\theta} d\theta = 0 \quad (44)$$

since correlations are zero outside the wake as well as at the centerline of the wake. The remaining shear stress term in Equation 21 is evaluated using the eddy viscosity concept.

$$\overline{u'w'} = \mu_T \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \quad (45)$$

Neglecting radial gradients gives,

$$\overline{u'w'} = \mu_T \left( \frac{\partial u}{\partial z} \right) \quad (46)$$

Hence, it is known that,

$$-\frac{d}{dz} \int_{\theta_c}^{\theta_o} \overline{(u'w')} d\theta = -\mu_T \frac{\partial}{\partial z} \int_{\theta_c}^{\theta_o} \frac{\partial u}{\partial z} d\theta \quad (47)$$

Using Equation 40 and Equation 47,

$$-\int_{\theta_c}^{\theta_o} \frac{\partial \overline{(u'w')}}{\partial z} d\theta = -\mu_T \frac{\partial}{\partial z} \left[ \frac{d}{dz} \int_{\theta_c}^{\theta_o} u d\theta + u_m \frac{\partial \theta_c}{\partial z} \right] \quad (48)$$

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$$= -\mu_T \frac{\partial}{\partial z} \left[ \frac{H}{r} \frac{d}{dz} (\delta u_m) + \frac{u_m}{r} \left( \frac{V_o}{W_o} - \frac{v_c}{w_c} \right) \right] \quad (49)$$

$$= -\mu_T \frac{H}{r} \frac{d^2}{dz^2} (\delta u_m) - \frac{\mu_T}{r} \frac{d}{dz} \left[ \frac{V_o u_m}{W_o} - \frac{v_c u_m}{W_o} + \frac{V_o w_c u_m}{W_o^2} \right] \quad (50)$$

Neglecting second order terms, Equation 50 becomes,

$$-\theta \int_c^o \frac{\partial}{\partial z} (\overline{u'w'}) d\theta = -\frac{\mu_T}{r} \left[ H \frac{d^2}{dz^2} (\delta u_m) + \frac{V_o}{W_o} \frac{d(u_m)}{dz} \right] \quad (51)$$

H is defined in Equation 58. Using a similar procedure the shear stress term in the tangential momentum equation can be proved to be,

$$\theta \int_c^o \frac{\partial}{\partial z} (\overline{v'w'}) d\theta = -\frac{\mu_T}{r} \left[ G \frac{d^2}{dz^2} (\delta v_c) + \frac{V_o}{W_o} \frac{d(v_c)}{dz} \right] \quad (52)$$

G is defined in Equation 58. The radial gradients of shear stress are assumed to be negligible.

Using Equations 39, 42, and 43, the remaining terms in Equations 21-24 are integrated across the wake (Reference 12). The radial, tangential, and axial momentum equations are now written as,

$$\begin{aligned} \frac{H}{r} S \frac{d(u_m \delta)}{dz} - \frac{(V_o + \Omega r)^2 \delta S}{r^2 W_o^2} + \frac{2\delta G(V_o + \Omega r) v_c S}{W_o r^2} = \\ = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial r} \frac{S}{W_o^2} - \frac{1}{\rho W_o^2} S \frac{\partial \theta}{\partial r} (P_c - P_e) + \frac{S P_e}{W_o^2 \rho} \left( \frac{\delta}{r^2} - \frac{d\delta}{r dr} \right) + \frac{\delta}{r} \frac{S}{W_o^2} \int_c^1 \left[ -\frac{\partial \overline{u'^2}}{\partial r} \right. \\ \left. - \frac{\overline{u'^2}}{r} + \frac{\overline{v'^2}}{r} \right] d\eta - \frac{S \mu_T}{W_o r} \left[ H \frac{d^2}{dz^2} (\delta u_m) + \frac{V_o}{W_o} \frac{du_m}{dz} \right] \end{aligned} \quad (53)$$

$$\begin{aligned} S \frac{\delta u_m}{W_o r} \left[ \frac{H}{dr} + \frac{V_o}{r} + 2\Omega \right] - \frac{GS}{r} \frac{d}{dz} (v_c \delta) = -\frac{1}{W_o^2 \rho} S (P_e - P_c) + \frac{\delta S}{W_o^2 r} \int_c^1 \left( -\frac{\partial \overline{v'^2}}{r \partial \theta} \right) d\eta \\ - \frac{S \mu_T}{W_o r} \left[ G \frac{d^2}{dz^2} (\delta v_c) + \frac{V_o}{W_o} \frac{dv_c}{dz} \right] \end{aligned} \quad (54)$$

$$\begin{aligned} \frac{dW_o}{dr} S \frac{\delta u_m H}{W_o r} - \frac{S}{r} \frac{d(w_c \delta)}{dz} F = - \frac{1}{W_o^2 \rho} S \frac{\partial \bar{P}}{\partial z} - \frac{1}{\rho r} S (P_c - P_e) \frac{V_o}{W_o^3} - \frac{P_e S}{W_o^2 \rho r} \frac{d\delta}{dz} \\ + \frac{\delta S}{W_o^2 r} \int_0^1 \left[ - \frac{\partial w'^2}{\partial z} \right] d\eta \end{aligned} \quad (55)$$

and continuity becomes

$$\frac{S v_c}{r} - \frac{S F}{r} \frac{d(\delta w_c)}{dz} + \frac{S \delta u_m H}{r^2} - \frac{S w_c V_o}{r W_o} = 0 \quad (56)$$

where  $w_c$ ,  $v_c$ , and  $u_m$  are nondimensional and from Reference 12,

$$G = \int_0^1 g(\eta) d\eta, \quad \frac{v}{v_c(z)} = g(\eta) \quad (57)$$

$$H = \int_0^1 h(\eta) d\eta, \quad \frac{u}{u_m(z)} = h(\eta) \quad (58)$$

$$F = \int_0^1 f(\eta) d\eta, \quad \frac{w}{w_c(z)} = f(\eta) \quad (59)$$

### 2.2.3 A relationship for pressure in Equations 53-55

The static pressures  $p_c$  and  $p$  are now replaced by known quantities and the four unknowns  $v_c$ ,  $u_m$ ,  $w_c$ , and  $\delta$ . This is done by using the  $n$  momentum equation ( $s$ ,  $n$  being the streamwise and principal normal directions respectively - Figure 1) of Reference 13,

$$\begin{aligned} U_r \frac{\partial U_n}{\partial r} + U_n \frac{\partial U_n}{\partial n} + U_s \frac{\partial U_n}{\partial s} + 2\Omega U_r \cos\beta - \frac{U_s^2}{R_c} + \frac{U_r U_n}{r} \cos^2\beta = \\ = - \frac{1}{\rho} \frac{\partial P^*}{\partial n} - \left\{ \frac{\partial}{\partial r} (\overline{u'_r u'_n}) + (1 + \cos^2\beta) \frac{\overline{u'_r u'_n}}{r} + \frac{\partial}{\partial n} \overline{u'^2_n} + \frac{\partial}{\partial s} (\overline{u'_n u'_s}) \right\} \end{aligned} \quad (60)$$

where

$$P^* = P - \frac{1}{2} \rho \Omega^2 r^2 \quad (61)$$

Non-dimensionalizing Equation 60 with  $\delta$  and  $U_{s0}$  and neglecting shear stress terms results in the following equation,

$$\begin{aligned}
& \frac{U_r}{U_{s_0}} \frac{\delta(U_n/U_{s_0})}{\delta(r/\delta)} + \frac{U_n}{U_{s_0}} \frac{\delta(U_n/U_{s_0})}{\delta(n/\delta)} + \frac{U_s}{U_{s_0}} \frac{\delta(U_n/U_{s_0})}{\delta(s/\delta)} + \frac{2\delta(\Omega r)U_r \cos \beta}{U_{s_0}^2 r} + \frac{U_r U_n}{U_{s_0}^2 (r/\delta)} \cos^2 \beta \\
& - \frac{U_s^2/U_{s_0}^2}{R_c/\delta} = - \frac{1}{\rho} \frac{\partial P^*}{\partial(n/\delta)} \frac{1}{U_{s_0}^2} - \frac{\partial(u_n'^2/U_{s_0}^2)}{\partial(n/\delta)} \quad (62)
\end{aligned}$$

An order of magnitude analysis is performed on Equation 62 using the following assumptions where  $\epsilon \ll 1$ ,

$$\begin{array}{ll}
n \sim \delta & U_n/U_{s_0} \sim \epsilon \\
s \sim \delta & U_r/U_{s_0} \sim 1 \\
\delta/r \sim \epsilon & \overline{u_n'^2}/U_{s_0}^2 \sim \epsilon \\
\Omega r/U_{s_0} \sim 1 & R_c \sim S \\
U_s/U_{s_0} \sim 1 & \delta/S \sim \epsilon
\end{array}$$

in the near wake region where the pressure gradient in the rotor wake is formed. The radius of curvature,  $R_c$  (Figure 1) is chosen to be of the order of blade spacing on the basis of the measured wake center locations shown in Figures 2 and 3.

The order of magnitude analysis gives,

$$\frac{U_r}{U_{s_0}} \frac{\partial(U_n/U_{s_0})}{\partial(r/\delta)} \sim \epsilon^2 \quad (63)$$

$$\frac{U_n}{U_{s_0}} \frac{\partial(U_n/U_{s_0})}{\partial(n/\delta)} \sim \epsilon^2 \quad (64)$$

$$\frac{U_s}{U_{s_0}} \frac{\partial(U_n/U_{s_0})}{\partial(s/\delta)} \sim \epsilon \quad (65)$$

$$\frac{2\delta(\Omega r)U_r \cos \beta}{U_{s_0}^2 r} \sim \epsilon \quad (66)$$

$$\frac{U_r U_n}{U_{s_0}^2 (r/\delta)} \cos^2 \beta \sim \epsilon^2 \quad (67)$$

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$$\frac{U_s^2/U_{s0}^2}{R_c/\delta} \sim \epsilon \quad (68)$$

$$\frac{\overline{\partial(u_n'^2/U_{s0}^2)}}{\partial(n/\delta)} \sim \epsilon \quad (69)$$

Retaining terms that are of order  $\epsilon$  in Equation 62 gives

$$\frac{U_s}{U_{s0}} \frac{\partial(U_n/U_{s0})}{\partial s} + \frac{2(\Omega r)U_r \cos \beta}{U_{s0}^2 r} - \frac{U_s^2/U_{s0}^2}{R_c} = -\frac{1}{\rho} \frac{\partial P^*}{\partial n} \frac{1}{U_{s0}^2} - \frac{\overline{\partial(u_n'^2/U_{s0}^2)}}{\partial n} \quad (70)$$

The first term in Equation 70 can be written as,

$$\frac{U_s}{U_{s0}} \frac{\partial(U_n/U_{s0})}{\partial s} = \frac{U_s}{U_{s0}} \frac{\partial((U_n/U_s)(U_s/U_{s0}))}{\partial s} \quad (71)$$

Since,

$$U_n/U_{s0} = \tan \alpha$$

where  $\alpha$  is the deviation from the TE outlet flow angle, Equation 71 becomes,

$$\begin{aligned} \frac{U_s}{U_{s0}} \frac{\partial(U_n/U_{s0})}{\partial s} &= \frac{U_s}{U_{s0}} \frac{\partial}{\partial s} \left[ \tan \alpha \frac{U_s}{U_{s0}} \right] \\ &= \frac{U_s}{U_{s0}} \left[ \frac{U_s}{U_{s0}} \sec^2 \alpha \frac{\partial \alpha}{\partial s} + \tan \alpha \frac{\partial(U_s/U_{s0})}{\partial s} \right] \\ &= \frac{U_s^2}{U_{s0}^2} \frac{1}{R_c} + \alpha \frac{U_s}{U_{s0}} \frac{\partial(U_s/U_{s0})}{\partial s} \end{aligned} \quad (72)$$

where

$$\frac{\partial \alpha}{\partial s} = \frac{1}{R_c}$$

and

$$\sec^2 \alpha \approx 1$$

$$\tan \alpha \approx \alpha$$

with  $\alpha$  small. Substituting Equation 72 and Equation 61 into Equation 70,

$$\alpha \frac{U_s}{U_{s0}} \frac{\partial(U_s/U_{s0})}{\partial s} + \frac{2(\Omega r)U_r \cos \beta}{U_{s0}^2 r} = - \frac{1}{\rho} \frac{\partial P}{\partial n} \frac{1}{U_{s0}^2} - \frac{\partial(\overline{u_n'^2}/U_{s0}^2)}{\partial n} \quad (73)$$

The integration across the rotor wake in the  $n$  direction (normal to streamwise) would result in a change in axial location between the wake center and the wake edge. In order that the integration across the wake is performed at one axial location, Equation 73 is transformed from the  $s, n, r$  system to  $r, \theta, z$ .

Expressing the streamwise velocity as

$$U_s^2 = V^2 + W^2$$

and

$$\frac{dz}{ds} = \cos \beta, \quad \frac{r d\theta}{ds} = \sin \beta$$

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Equation 73 becomes,

$$\begin{aligned} \alpha \left( \frac{V^2 + W^2}{V_o^2 + W_o^2} \right)^{1/2} \left[ \cos \beta \frac{\partial}{\partial z} \left( \frac{V^2 + W^2}{V_o^2 + W_o^2} \right)^{1/2} + \frac{\sin \beta}{r} \frac{\partial}{\partial \theta} \left( \frac{V^2 + W^2}{V_o^2 + W_o^2} \right)^{1/2} \right] + \frac{2(\Omega r)U_r \cos \beta}{V_o^2 + W_o^2} = \\ = - \frac{1}{\rho} \frac{\cos \beta}{r} \frac{P_e - P_c}{V_o^2 + W_o^2} - \cos \beta \frac{\partial}{r \partial \theta} \left( \frac{\overline{u_n'^2}}{U_{s0}^2} \right) - \sin \beta \frac{\partial}{\partial z} \left( \frac{\overline{u_n'^2}}{U_{s0}^2} \right) \end{aligned} \quad (74)$$

Expressing radial, tangential, and axial velocities in terms of velocity defects  $u, v$ , and  $w$  respectively, Equation (74) can be written as,

$$\begin{aligned} - \frac{1}{2} \frac{\alpha r \cos \beta}{V_o^2 + W_o^2} \left[ \int_{\theta_c}^{\theta_o} \left( \frac{\partial v^2}{\partial z} + V_o \frac{\partial v}{\partial z} \right) d\theta + \int_{\theta_c}^{\theta_o} \left( \frac{\partial w^2}{\partial z} + W_o \frac{\partial w}{\partial z} \right) d\theta \right] - \frac{1}{2} \frac{\alpha}{V_o^2 + W_o^2} \sin \beta \left[ \int_{\theta_c}^{\theta_o} \left( \frac{\partial v^2}{\partial \theta} + \right. \right. \\ \left. \left. + V_o \frac{\partial v}{\partial \theta} \right) d\theta + \int_{\theta_c}^{\theta_o} \left( \frac{\partial w^2}{\partial \theta} + W_o \frac{\partial w}{\partial \theta} \right) d\theta \right] + \cos \beta \frac{2(\Omega r)r}{V_o^2 + W_o^2} \int_{\theta_c}^{\theta_o} u d\theta = \\ = - \frac{1}{V_o^2 + W_o^2} \frac{\cos \beta}{\rho} (P_e - P_c) - \cos \beta \int_{\theta_c}^{\theta_o} \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{\overline{u_n'^2}}{U_{s0}^2} \right) d\theta - \sin \beta \int_{\theta_c}^{\theta_o} \frac{\partial}{\partial z} \left( \frac{\overline{u_n'^2}}{U_{s0}^2} \right) d\theta \end{aligned} \quad (75)$$

Using the integration technique given in Equations 39-43, Equation 75 becomes,

$$\begin{aligned}
& - \frac{W_o^2}{2} \alpha \left[ \frac{d(v_c^2 \delta)}{dz} G_1 + V_o \frac{d(v_c \delta)}{dz} G + (v_c^2 + V_o v_c) \left( \frac{V_o - v_c}{W_o - w_c} \right) + \frac{d(w_c^2 \delta)}{dz} F_1 + W_o \frac{d(w_c \delta)}{dz} F \right. \\
& \quad \left. + (w_c^2 + w_c W_o) \left( \frac{V_o - v_c}{W_o - w_c} \right) \right] + \frac{W_o^2}{2} \tan \beta [v_c^2 + \frac{V_o}{W_o} v_c + w_c^2 + w_c] \\
& \quad + 2W_o (\Omega r) \delta u_m H + \frac{(W_o^2 + V_o^2)}{r} \theta_c^{\theta_o} \frac{\partial}{\partial \theta} \left( \frac{u_n'^2}{U_{s_o}^2} \right) d\theta + \frac{W_o^2 + V_o^2}{r} \theta_c^{\theta_o} \frac{\partial}{\partial z} \left( \frac{u_n'^2}{U_{s_o}^2} \right) d\theta \\
& = - \frac{1}{\rho} (P_e - P_c) \tag{76}
\end{aligned}$$

where,

$$G_1 = \int_0^1 g^2(\eta) d\eta, \quad g(\eta) = \frac{v}{v_c(z)} \tag{77}$$

$$F_1 = \int_0^1 f^2(\eta) d\eta, \quad f(\eta) = \frac{w}{w_c(z)} \tag{78}$$

Non-dimensionalizing Equation 76 using velocity  $W_o$  gives,

$$\begin{aligned}
& - \frac{1}{2} \alpha \left[ \frac{d(v_c^2 \delta)}{dz} G_1 + \frac{V_o}{W_o} \frac{d(v_c \delta)}{dz} G + (v_c^2 + \frac{V_o}{W_o} v_c) \left( \frac{V_o - v_c}{W_o - w_c} \right) + \frac{d(w_c^2 \delta)}{dz} F_1 + \frac{d}{dz} (w_c \delta) F \right. \\
& \quad \left. + (w_c^2 + w_c) \left( \frac{V_o - v_c}{W_o - w_c} \right) \right] + \frac{1}{2} \alpha \tan \beta [v_c^2 + \frac{V_o}{W_o} v_c + w_c^2 + w_c] \\
& \quad + 2 \frac{(\Omega r)}{W_o} u_m \delta H + (1 + \frac{V_o^2}{W_o^2}) \theta_c^{\theta_o} \frac{\partial}{\partial \theta} \left( \frac{u_n'^2}{U_{s_o}^2} \right) d\theta + (1 + \frac{V_o^2}{W_o^2}) \theta_c^{\theta_o} \frac{\partial}{\partial z} \left( \frac{u_n'^2}{U_{s_o}^2} \right) d\theta \\
& = - \frac{1}{W_o^2 \rho} (P_e - P_c) \tag{79}
\end{aligned}$$

Note,  $v_c$ ,  $w_c$ , and  $u_m$  are now normalized by  $W_o$  in Equation 79.

#### 2.2.4 Solution of the momentum integral equation

Equation 79 represents the variation of static pressure in the rotor wake as being controlled by Coriolis force, centrifugal force, and normal gradients of turbulence intensity. From the measured curvature of the rotor wake velocity

centerline shown in Figures 2 and 3, the radius of curvature is assumed negligible. This is in accordance with far wake assumptions previously made in this analysis. Thus, the streamline angle ( $\alpha$ ) at a given radius is small and Equation 79 becomes,

$$\frac{2\delta(\Omega r)u_m}{W_o} + \left(1 + \frac{V_o^2}{W_o^2}\right) \theta_c^{\theta_o} \frac{\partial}{\partial \theta} \left( \frac{\overline{u_n'^2}}{U_{s_o}^2} \right) d\theta + \left(1 + \frac{V_o^2}{W_o^2}\right) \theta_c^{\theta_o} \frac{\partial}{\partial z} \left( \frac{\overline{u_n'^2}}{U_{s_o}^2} \right) d\theta = - \frac{(P_e - P_c)}{W_o^2 \rho} \quad (80)$$

Substitution of Equation 80 into Equations 53, 54, and 55 gives a set of four equations (continuity and r,  $\theta$ , and z momentum) which are employed in solving for the unknowns  $\delta$ ,  $u_m$ ,  $v_c$ , and  $w_c$ . All intensity terms are substituted into the final equations using known quantities from the experimental phase of this investigation.

A fourth order Runge-Kutta program is to be used to solve the four ordinary differential equations given above. Present research efforts include working towards completion of this programming.

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### 3. EXPERIMENTAL DATA

#### 3.1 Experimental Data From the AFRF Facility

Rotor wake data has been taken with the twelve bladed un-cambered rotor described in Reference 13 and with the recently built nine bladed cambered rotor installed. Stationary data was taken with a three sensor hot wire for both the nine bladed and twelve bladed rotors so that mean velocity, turbulence intensity, and turbulent stress characteristics of the rotor wake could be determined. Rotating data using a three sensor hot wire probe was taken with the twelve bladed rotor installed so that the desired rotor wake characteristics listed above would be known.

##### 3.1.1 Rotating probe experimental program

As described in Reference 1, the objective of this experimental program is to measure the rotor wakes in the relative or rotating frame of reference. Table 1 lists the measurement stations and the operating conditions used to determine the effect of blade loading (varying incidence and radial position) and to give the decay characteristics of the rotor wake. In Reference 1 the results from a preliminary data reduction were given. A finalized data reduction has been completed which included the variation of  $E_0$  due to temperature changes, wire aging, and other changes in the measuring system which affected the calibration characteristics of the hot wire. The results from the final data reduction at  $r/r_t = 0.720$ ,  $i = 10^\circ$ , and  $x/c = 0.021$  and  $0.042$  are presented here. Data at other locations will be presented in the final report.

Static pressure variations in the rotor wake were measured for the twelve bladed rotor. The probe used was a special type which is insensitive to changes in the flow direction (Reference 1). A preliminary data reduction of the static pressure data has been completed and some data is presented here.



Table 1

Experimental Variables for Rotating Hot Wire and  
Static Pressure Measurements

|                                                                      |                                          |
|----------------------------------------------------------------------|------------------------------------------|
| Incidence                                                            | 0°, 10°                                  |
| Radius ( $r/r_t$ )                                                   | 0.720 and 0.860                          |
| Axial distance from trailing edge                                    | 1/8", 1/4", 1/2", 1", and 1 5/8"         |
| Axial distance $x/c$ (non-dimensionalized<br>w.r.t. the local chord) | 0.021, 0.042, 0.083, 0.167, and<br>0.271 |

The output from the experimental program included the variation of three mean velocities, three r.m.s. values of fluctuating velocities, three correlations, and static pressures across the rotor wake at each wake location. In addition, a spectrum analyzer was utilized to derive the spectrum of fluctuating velocities from each wire.

This set of data provides properties of about sixteen wakes, each were defined by 20 to 30 points.

### 3.1.1.1 Interpretation of the results from three-sensor hot-wire final data reduction

Typical results for  $r/r_t = 0.72$  are discussed below. This data is graphed in Figure 4 through 24. The abscissa shows the tangential distance in degrees. Spacing of the blade passage width is 30°.

#### (a) Streamwise component of the mean velocity, US

Variation of  $US/US_0$  downstream of the rotor wake, Figures 4 and 5, indicates the decay characteristics of the rotor wake. The velocity defect at the wake centerline  $(US)_d/US_0$  is about 0.45 at  $x/c = 0.021$  and decays to 0.56 at  $x/c = 0.042$ .

The total velocity defect  $(Qd)_c/Q_0$  at the centerline of the wake is plotted with respect to distance from the blade trailing edge in the streamwise direction in Figure 6 and is compared with other data from this experimental program. Also presented is other Penn State data, data due to Ufer [14], Hanson [15], isolated

airfoil data from Preston and Sweeting [16], and flat plate wake data from Chevray and Kovasznay [17]. In the near wake region decay of the rotor wake is seen to be faster than the isolated airfoil. In the far wake region the rotor, isolated airfoil, and flat plate wakes have all decayed to the same level.

Figures 4 and 5 also illustrate the asymmetry of the rotor wake. This indicates that the blade suction surface boundary layer is thicker than the pressure surface boundary layer. The asymmetry of the rotor wake is not evident at large  $x/c$  locations indicating rapid spreading of the wake and mixing with the free stream.

(b) Normal component of mean velocity,  $U_N$

Variation of normal velocity at both  $x/c = 0.021$  and  $x/c = 0.042$  indicate a deflection of the flow towards the pressure surface in the wake, Figures 7 and 8. The deflection is seen to decay from  $x/c = 0.021$  to  $x/c = 0.042$ . This decay results from flow in the wake turning to match that in the free stream.

(c) Radial component of mean velocity,  $U_R$

The behavior of the radial velocity in the wake is shown in Figures 9 and 10. The radial velocity at the wake centerline is relatively large at  $x/c = 0.021$ , and decays by 50 percent by  $x/c = 0.042$ . Decay of the radial velocity is much slower downstream of  $x/c = 0.042$ .

(d) Turbulent intensity, FWS, FWN, FWR

Variation of FWS, FWN, and FWR across the rotor wake at  $x/c = 0.021$  and  $x/c = 0.042$  as shown in Figures 11 and 12, Figures 13 and 14, and Figures 15 and 16, respectively. Intensity in the  $r$  direction is seen to be larger than the intensities in both the  $s$  and  $n$  directions. Figures 15 and 16 show the decay of FWR at the wake centerline by 50% over that measured in the free stream. Decay at the wake centerline is slight between  $x/c = 0.021$  and  $x/c = 0.047$  for both

FWS and FWN. At  $x/c = 0.167$ , FWS, FWN, and FWR have all decayed to close to that in the free stream.

This final data reduction indicates large gradients of,

$$\frac{\partial(FWR)}{\partial z}, \quad \frac{\partial(FWR)}{\partial \theta}$$

with much slower decays of FWS and FWN in the near wake. To correctly predict the flow in this region, the terms containing large gradients of FWR must be retained.

(e) Turbulent stress, B(4), B(5), and B(6)

Stresses in the s, n, r directions are shown in Figures 17 and 18, Figures 19 and 20, and Figures 21 and 22, respectively. Turbulent stress B(4) is seen to be negative on pressure surface side of the wake centerline, positive on the suction surface side, and zero at the wake centerline. Decay of B(4) from  $x/c = 0.021$  to  $x/c = 0.042$  is large. Beyond  $x/c = 0.042$  decay of turbulent stress B(4) is much slower. Turbulent stress B(5) is small compared to the levels measured for B(4) and B(6). Turbulent stress B(6) is 0.015 at  $x/c = 0.021$  and decays at .05 of the peak seen on the pressure surface side in the rotor wake at  $x/c = 0.042$ . Decay is much slower downstream of  $x/c = 0.042$ .

(f) Resultant stress, RESSTR

Variation of resultant stress across the rotor wake is shown in Figures 23 and 24 for  $x/c = 0.021$  and  $x/c = 0.042$ . Resultant stress is highest on the pressure surface side of the wake centerline. This peak decays from a resultant stress level of 0.017 at  $x/c = 0.021$  to a level of 0.005 at  $x/c = 0.042$ . At  $x/c = 0.271$ , RESSTR has decayed to close to its free stream value.

### 3.1.1.2 Interpretation of the results from static pressure preliminary data reduction

Variation of static pressure across the rotor wake is shown in Figure 25 through 27 at  $r/r_t = 0.721$  for  $x/c = 0.021$ , 0.083, and 0.271. Local static

pressure in the wake is non-dimensionalized with wake edge static pressure,  $P_e$ , on the pressure surface side of the wake. Large gradients of  $P/P_e$  is clearly seen at all axial locations. This indicates that when theoretically predicting the rotor wake that terms such as,

$$\frac{\partial P}{\partial \theta}, \quad \frac{\partial P}{\partial z}$$

must be retained. However, the variation in static pressure shown in Figure 25 through 27 is not clearly understood. At a larger radial position ( $r/r_t = 0.860$ ), the static pressure variation shown in Figure 28 was measured at  $x/c = 0.021$ . Comparison of Figures 25 and 28 does not show a similar variation of  $P/P_e$  across the wake as would be expected. For this reason, the results shown here are only preliminary as the data reduction procedure used in calculating  $P$  is presently being checked. Therefore, the only reliable conclusion that can be drawn from this preliminary data reduction is that gradients of static pressure do exist in the rotor wake.

### 3.1.2 Stationary probe experimental program

Using a three-sensor hot wire probe, stationary data was recorded using the technique described in Reference 13. Measurement locations and the operating conditions used in the measurement of the wake of a 12 bladed uncambered rotor is described in Reference 1. With a 9 bladed cambered rotor installed, stationary data was taken at three operation conditions in both the near and far wake regions. Axial distance was varied from  $x/c = 0.042$  to about 1.2 chords downstream at ten radial positions from hub to tip ( $r/r_t = .465, .488, .512, .535, .628, .721, .814, .916, .940, \text{ and } .963$ ). This data will indicate the effect of end wall flows and the hub wall boundary layer on the rotor wake. To determine the centrifugal force effect on the rotor wake, data was recorded at design conditions with two rotor RPM's (RPM = 1753 and 1010). To indicate the effect of loading, the data was also taken at an additional off-design condition at each measurement location.

The nine bladed data described here and the previously taken twelve bladed data will be reduced when the data processing computer programming has been completed

### 3.2 Experimental Program at the Axial Flow Compressor Facility

Rotor wake measurements to be carried out at the axial flow compressor facility consist of both the rotating and the stationary hot wire measurements along with a static/stagnation pressure survey across the wake. The purpose of this investigation is to study the following:

1. Wake development and decay as it travels downstream.
2. Nature of wake along the radial direction.
3. Nature of wake in the hub and annulus wall boundary layers.
4. Effect of blade loading on the wake.
5. Effect of IGV wakes on the rotor wake.
6. Effect of inlet distortions on the wake.
7. Comparison of measurements in stationary and rotating frame of reference.

For the pressure survey a special type of static/stagnation probe is being fabricated. It is designed to be insensitive to direction changes.

Presently, the rotating hot wire measurements are being carried out. Using the traverse gear, which was modified to suit the present requirements, the three sensor hot wire is being traversed across the wake. A sketch of the modified tranverse gear is shown in Figure 29. The output which is in the form of three D.C. voltages, three r.m.s. voltages and three correlating voltages are being processed. Some preliminary results of the rotor wake, measured at radius = 9.448" ( $R/R_t = .45$ ) and 1.75 inches axial distance downstream of the trailing edge (corresponds to 0.3 times chord length) at the design operating condition ( $\phi = 0.54$ ) are shown in Figures 30 and 31. The width of the passage is  $18^\circ$ . The streamwise velocity  $U_s$ , shown in Figure 30, shows a rather slower decay of the wake in this configuration of heavily loaded rotor. The radial velocities, shown

in Figure 31, are also higher than those reported in previous cases. The turbulence intensities (not shown here) are also found to be high.

#### 4. SYNTHESIS AND CORRELATION OF ROTOR WAKE DATA

##### 4.1 Similarity rule in the rotor wake

There is sufficient experimental evidence about similarity of mean velocity profiles in successive sections of wake flows in the downstream direction. The free-turbulent-flow regions are relatively narrow, with a main-flow velocity much greater than the transverse component. The spacial changes in the main-flow direction are much smaller than the corresponding changes in the transverse direction. Furthermore, the turbulence flow pattern moving downstream is strongly dependent on its history. These facts make similarity probable in wake flows. The similarity rule is examined for rotor wake flows. The data due to Raj and Lakshminarayana [13] and Schmidt and Okiishi [18] as well as those present in Section 3 are examined.

##### 4.1.1 Data of Raj and Lakshminarayana [13]

The maximum mean velocity difference is used as a velocity scale to make all the velocities non-dimensional. The so-called "half-value" distance which is the distance from the axis of symmetry to the location where the mean velocity defect is half the maximum value, is used as a length scale.

Similarity in axial velocity profile was discussed in Reference 12 and will not be repeated here.

Due to imbalance between centrifugal force and static pressure in the radial direction, the radial velocity component is formed. In the blade boundary layer, the radial velocity profile is determined by the above mentioned imbalance of forces acting on the fluid particle. The experimental data of rotor wake shows that there are two dominant radial velocity scales. One is dominant in the center region and the other is dominant in the outer region of the wake. Figure 32 shows general profile of radial velocity and notations used. In the center region ( $0 \leq |y| < |L_p|$  or  $|L_s|$ ) the velocity is non-dimensionalized with  $(\Delta u_{r_m})_p$  or  $(\Delta u_{r_m})_s$ . In the outer region the velocity is non-dimensionalized by  $(\Delta u_r)_p$  or

$(\Delta u_p)_s$ . The length  $L_p$  or  $L_s$  is used for non-dimensionalizing length scale.

Figure 33 and Figure 34 show typical radial velocity distribution with the above mentioned velocity and length scales. In the center region, the profile is nearly linear. In the outer region, the velocity is very small, and hence the large scatter in the data.

#### 4.1.2 Data of Schmidt and Okiishi [18]

The velocity profiles with the same corresponding velocity and length scales are shown in Figure 35 and Figure 36 for the Iowa State data. Better agreement with theoretical distribution in axial velocity component is shown. Good agreement with theoretical Gaussian distribution is also shown for the absolute tangential velocity profile plotted in Figure 37.

With regard to the radial velocity component, plotted in Figure 38, the new velocity and length scale defined earlier are used successfully. The bell-shaped velocity profile curves are found in axial and tangential velocity components. But for radial velocity components, different length and velocity scales are necessary and the velocity profile is nearly linear near the center region.

#### 4.1.3 Rotating probe data from the AFRF facility

The data reported in Section 3 of this report is plotted in Figures 39 through 50 in a non-dimensional form using the characteristic length scale ( $L_p$  and  $L_s$ ) and velocity ( $w_c$ ,  $u_c$  and  $u_m$ ) discussed earlier. These results are from a twelve bladed rotor operating at  $0^\circ$  and  $10^\circ$  incidence and taken with a rotating triaxial probe.

The axial velocity profile, shown plotted in Figure 39 and 40, seem to follow the Gauss' function ( $e^{-0.693\eta^2}$ ). The tangential velocity defect, shown in Figure 41 and 42, show the same trend. Even the radial velocity seem to follow this trend (Figure 43). The nature of these radial velocity profiles are different from those reported earlier (Figures 33, 34, and 38). Further work with regard to the existence of similarity in radial velocity is presently underway.



The radial, axial and tangential component of turbulence intensity profiles at various incidence, radial and axial locations are shown in Figures 44 through 49. The intensities are plotted as

$$\frac{\overline{u'^2}}{\overline{u_c'^2}} - \frac{\overline{u_e'^2}}{\overline{u_c'^2}} \text{ vs } \eta_p \text{ or } \eta_s$$

The intensities are normalized with respect to the extrapolated peak intensity near the center and the corresponding value in the free stream is subtracted from the local value. All the data seem to follow the Gauss' function

#### 4.2 Fourier Decomposition of the Rotor Wake

The rotor wake can be conveniently represented in terms of a Fourier Series. The velocity at a position  $(r, \theta, z)$  can be represented as:

$$u_{D_N}(r, \theta, z) = A_0 + \sum_{n=1}^{\infty} [A_n(r, z) \cos \frac{n\theta}{\theta_L} + B_n(r, z) \sin \frac{n\theta}{\theta_L}] \quad (81)$$

where

$$u_{D_n} = \text{normalized velocity defect} = \frac{u_D}{u_{D_L}} = \frac{u_{\max} - u}{u_{\max} - u_{\min}}$$

$$\theta_L = \text{distance between wake centerline and the location where the } u_D = \frac{1}{2} \text{ (wake width)}$$

$$u_{\max} = \text{maximum velocity in the wake}$$

$$u_{\min} = \text{centerline velocity in the wake}$$

$$u = \text{velocity at any point}$$

In the present study, the wake data from the axial flow research fan reported in Reference 13 is analyzed by Fourier decomposition. The method is based on the recursive technique of Ralston [19]. In the analysis, the velocity defect was normalized by the wake centerline defect and all the angles by the angle at the half wave width in Equation 81.  $A_0$  represents half the average of the wake velocity in the given interval.

Now

$$A_n = \frac{\epsilon_n}{2\pi} \int_0^{2\pi} \frac{u_D}{u_{D_L}} \cos \frac{n\theta}{\theta_L} d\theta$$

where  $\epsilon_n$  = Neumann's factor (= 1 for  $n = 0$ ; = 2 for  $n > 0$ )

$2\pi$  = interval of integration (in the analysis only the wake portion is considered).

$$A_0 = \frac{1}{2\pi} \frac{1}{u_{D_c}} \int_0^{2\pi} u_D d\theta$$

= average over the interval

Similarly  $A_n$  and  $B_n$  are calculated. As the coefficients represent the normalized velocity defect, it should stay constant if similarity exists. Figures 50 to 56 shows the coefficients plotted against distance from the trailing edge normalized by the blade chord. It is obvious that there is considerable scatter in the coefficients indicating the scatter in the data. Figure 56 shows the scatter of the coefficients for the first four harmonics, which were found to be the dominant ones. The values of  $B_n$ , the amplitude of the sine component of the series was found to be extremely small indicating the axisymmetric nature of the wake, where all the distances were normalized by the respective length scale and the local velocity defect by the centerline defects. The Fourier series matches quite well with the data points, when only the first four harmonics were considered. Figures 57 and 59 shows how closely the two curves match. The correlation coefficient for the entire wake was found to be around 0.9988.

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## APPENDIX I

## Nomenclature

|                                                        |                                                                                                                                              |
|--------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| $B(4), B(5), B(6)$                                     | = r.m.s. turbulent stress in the streamwise, normal, and radial directions respectively (normalized by local resultant dynamic head)         |
| $c$                                                    | = Chord length                                                                                                                               |
| $c_l$                                                  | = Lift coefficient                                                                                                                           |
| $c_1, c_\epsilon, c_{\epsilon 1}, c_{\epsilon 2}, c_c$ | = Universal constants                                                                                                                        |
| $F, G, H$                                              | = See Equations 57-59                                                                                                                        |
| $F_1, G_1$                                             | = See Equations 77 and 78                                                                                                                    |
| $FWS, FWN, FWR$                                        | = r.m.s. turbulent intensity in the streamwise, normal, and radial directions, respectively (normalized w.r.t. the local resultant velocity) |
| $g_{ij}$                                               | = Metric tensor                                                                                                                              |
| $i$                                                    | = Incidence                                                                                                                                  |
| $k$                                                    | = Kinetic turbulent energy $[\frac{1}{2}(\overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2})]$                                           |
| $m$                                                    | = Passage-mean mass flow rate                                                                                                                |
| $L_p, L_s$                                             | = Wake width at half depth on the pressure and suction side respectively                                                                     |
| $p$                                                    | = Static pressure                                                                                                                            |
| $p_c$                                                  | = Wake center static pressure                                                                                                                |
| $p_e$                                                  | = Wake edge static pressure                                                                                                                  |
| $\overline{p}$                                         | = Mean static pressure at y,r plane (in momentum integral analysis this is wake average pressure)                                            |
| $p'$                                                   | = Fluctuating component of static pressure                                                                                                   |
| $p''$                                                  | = Correction part of static pressure                                                                                                         |
| $p^*$                                                  | = Reduced pressure                                                                                                                           |
| $Q$                                                    | = Total relative mean velocity                                                                                                               |
| $Q_d$                                                  | = Defect in total relative mean velocity                                                                                                     |
| $RESSTR$                                               | = Total resultant stress $\sqrt{[B(4)]^2 + [B(5)]^2 + [B(6)]^2}$                                                                             |
| $R_c$                                                  | = Local radius of curvature of streamwise flow in wake.                                                                                      |

|                                                     |                                                                                                                                       |
|-----------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| $r, \theta, z$                                      | = Rotating cylindrical coordinate system (radial, tangential, and axial directions, respectively, $z = 0$ is trailing edge, Figure 1) |
| $S$                                                 | = Blade spacing                                                                                                                       |
| $s, n, r$                                           | = Streamwise, normal, and radial directions, respectively (Figure 1)                                                                  |
| $s'_{ij}$                                           | = Strain rate fluctuations $[\frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})]$                     |
| $u, v, w,$                                          | = Relative radial, tangential, and axial mean velocities, respectively (Figure 1)                                                     |
| $U_S, U_N, U_R$                                     | = Relative streamwise, normal, and radial mean velocities, respectively                                                               |
| $U_S$                                               | = Relative streamwise mean velocity in free stream                                                                                    |
| $U_s, U_n, U_r$                                     | = Relative streamwise normal and radial mean velocities, respectively (Figure 1)                                                      |
| $U_i$                                               | = Component of mean velocity                                                                                                          |
| $u, v, w$                                           | = Defect in radial, relative tangential and axial velocities, respectively, in the wake                                               |
| $u_i$                                               | = Components of turbulent velocity                                                                                                    |
| $u_{j,k}, v_{j,k}, w_{j,k}$                         | = Value of $u$ , $v$ , and $w$ at node coordinate $(j,k)$                                                                             |
| $u_m$                                               | = Maximum radial velocity away from the centerline normalized by $W_o$                                                                |
| $u', v', w'$                                        | = Turbulence velocities in the radial, tangential, and axial directions, respectively (Figure 1)                                      |
| $\overline{u'v'}, \overline{u'w'}, \overline{v'w'}$ | = Turbulence velocity correlation                                                                                                     |
| $u^*, v^*, w^*$                                     | = Intermediate value of $u$ , $v$ , $p$ during finite different calculation                                                           |
| $u'_s, u'_n, u'_r$                                  | = Turbulent velocities in the streamwise, normal and radial directions, respectively (Figure 1)                                       |
| $V_o, W_o$                                          | = Relative tangential and axial velocity, respectively, outside of the wake                                                           |
| $V_\theta$                                          | = Streamline velocity in the definition of Richardson number                                                                          |
| $v_c, w_c$                                          | = Defect in tangential (relative) and axial velocity, respectively, at the wake centerline normalized by $W_o$                        |
| $x$                                                 | = Axial distance                                                                                                                      |

|                                |                                                                          |
|--------------------------------|--------------------------------------------------------------------------|
| $\Delta x, \Delta y, \Delta z$ | = Mesh size in x, y and r direction, respectively                        |
| $\Omega$                       | = Angular velocity of the rotor                                          |
| $\alpha$                       | = Angle between streamwise and blade stagger directions (Figure 1)       |
| $\beta$                        | = Angle between axial and streamwise directions (Figure 1)               |
| $\delta$                       | = Semi wake thickness $[r(\theta_c - \theta_o)]$                         |
| $\delta_{ij}$                  | = Kronecker-delta                                                        |
| $\varepsilon$                  | = Turbulence dissipation                                                 |
| $\varepsilon_{ij}$             | = Dissipation tensor                                                     |
| $\varepsilon_{ijk}$            | = Alternating tensor                                                     |
| $\theta_c, \theta_o$           | = Angular coordinate of the wake centerline and wake edge, respectively  |
| $\eta_s, \eta_p$               | = Wake transverse distances normalized by $L_s$ and $L_p$ , respectively |
| $\lambda$                      | = Blade stagger                                                          |
| $\mu_T$                        | = Turbulent eddy viscosity                                               |
| $\nu$                          | = Kinetic viscosity                                                      |
| $\nu_t$                        | = Effective kinetic viscosity                                            |
| $\rho$                         | = Density                                                                |
| $\phi$                         | = Mass averaged flow coefficient (based on blade tip speed)              |

#### Subscript

|     |                                 |
|-----|---------------------------------|
| o,e | = Values outside the wake       |
| c   | = Values at the wake centerline |
| -   | = Wake averaged values          |
| t   | = Tip                           |
| 2D  | = Two dimensional cascade       |

## APPENDIX II

## Numerical Analysis

A.1 Basic Equations

Some of the notations and technique used are illustrated in Figure 60.

Other notations are defined in Appendix I.

Continuity

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$$

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Momentum equation in rotating coordinate system.

x direction (Figure 60)

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial r} + 2\Omega W \sin\beta = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \left\{ \frac{\partial}{\partial x} \overline{u'^2} + \frac{\partial}{\partial y} \overline{u'v'} + \frac{\partial}{\partial r} \overline{u'w'} \right\}$$

y direction (Equation 60)

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial r} - 2\Omega W \cos\beta = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \left\{ \frac{\partial}{\partial x} \overline{u'v'} + \frac{\partial}{\partial y} \overline{v'^2} + \frac{\partial}{\partial r} \overline{v'w'} \right\}$$

r direction (Figure 60)

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial r} + 2\Omega V \cos\beta - 2\Omega U \sin\beta - r\Omega^2 = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \left\{ \frac{\partial}{\partial x} \overline{u'w'} + \frac{\partial}{\partial y} \overline{v'w'} + \frac{\partial}{\partial r} \overline{w'^2} \right\} \quad -$$

where x is streamline direction and r is radial direction and y is binormal direction. The coordinate frame is chosen as Cartesian at each marching step.

A.2 Equations for the Numerical Analysis, Including Turbulence Closure

The parabolic-elliptic Navier-Stokes equations at each marching step with x as the streamline direction are given below. Lower order terms in the equations of motion (Section A1 above) are dropped resulting in the following equations.

x direction:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial r} + 2\Omega W \sin\beta = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \left\{ \frac{\partial}{\partial y} \overline{u'v'} + \frac{\partial}{\partial r} \overline{u'w'} \right\}$$

y direction:

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial r} - 2\Omega W \cos\beta = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \left\{ \frac{\partial}{\partial y} \overline{v'^2} + \frac{\partial}{\partial r} \overline{v'w'} \right\}$$

r direction

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial r} + 2\Omega v \cos\beta - 2\Omega U \sin\beta - r\Omega^2 = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \left\{ \frac{\partial}{\partial y} \overline{v'w'} + \frac{\partial}{\partial r} \overline{w'^2} \right\}$$

Here a turbulence model for the Reynolds stresses is introduced.

$$\overline{u'_i u'_j} = \mu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}$$

Then the momentum equation becomes

x momentum

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial r} + 2\Omega W \sin\beta = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_t \left\{ \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 V}{\partial x \partial y} + \frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 W}{\partial x \partial r} \right\}$$

y momentum

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial r} - 2\Omega W \sin\beta = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu_t \left\{ 2 \frac{\partial^2 V}{\partial y^2} - \frac{2}{3} \frac{\partial k}{\partial y} + \frac{\partial^2 V}{\partial r^2} + \frac{\partial^2 W}{\partial r \partial y} \right\}$$

r momentum

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial r} + 2\Omega v \cos\beta - 2\Omega U \sin\beta - r\Omega^2 = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu_t \left\{ \frac{\partial^2 W}{\partial r^2} - \frac{2}{3} \frac{\partial k}{\partial r} + \frac{\partial^2 V}{\partial r \partial y} + \frac{\partial^2 W}{\partial y^2} \right\}$$

### A.3 Marching to New x Station with Alternating Directions Implicit Method

The periodic boundary condition is imposed on boundary surfaces where  $y =$  constant. Therefore, the first step is r-direction implicit and the second step is y-direction implicit.

#### A.3.1 r implicit step

##### A.3.1.1 Equation for new streamwise velocity components at the new station

$$U_{j,k} \frac{u_{j,k} - U_{j,k}}{\Delta x} + V_{j,k} \frac{U_{j+1,k} - U_{j-1,k}}{2\Delta y} + W_{j,k} \frac{u_{j,k+1} - u_{j,k-1}}{2\Delta r} + 2\Omega W_{j,k} \sin\beta = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_t \left\{ \frac{U_{j-1,k} - 2U_{j,k} + U_{j+1,k}}{(\Delta y)^2} + \frac{u_{j,k-1} - 2u_{j,k} + u_{j,k+1}}{(\Delta r)^2} \right\}$$

$$\begin{aligned}
& + \frac{1}{4\Delta x \Delta y} (v_{j+1,k+1} + v_{j-1,k-1} - v_{j+1,k-1} - v_{j-1,k+1}) \\
& + \frac{1}{4\Delta x \Delta r} (w_{j+1,k+1} + w_{j-1,k-1} - w_{j+1,k-1} - w_{j-1,k+1}) \}
\end{aligned}$$

This equation can be rearranged as

$$A u_{j,k+1} + B u_{j,k} + C u_{j,k-1} = D$$

This system of linear equations with boundary condition can be solved easily with standard AD1 scheme. See Figure 60 for the definition of U, V, W and u, v, w.

#### A.3.1.2 Equations for first approximation for v,w in the new x station

With the upper station value of static pressure, v and w are calculated. The finite difference form of v momentum equation is as follows:

$$\begin{aligned}
U_{j,k} \frac{v_{i,k} - v_{j,k}}{\Delta n} + v_{j,k} \frac{v_{j+1,k} - v_{j-1,k}}{2\Delta y} + w_{j,k} \frac{v_{j,k+1} - v_{j,k-1}}{2\Delta r} - 2\Omega w_{j,k} \cos\beta \\
= - \frac{1}{\rho} \frac{P_{j+1,k} - P_{j-1,k}}{2\Delta y} + v_t \{ 2.0 \frac{v_{j+1,k} - 2v_{j,k} + v_{j-1,k}}{(\Delta y)^2} \\
- \frac{2}{3} \frac{K_{j+1,k} - K_{j,k}}{\Delta y} + \frac{v_{j,k+1} - 2v_{j,k} + v_{j,k-1}}{(\Delta r)^2} \\
+ \frac{1}{4\Delta y \Delta y} (w_{j+1,k+1} + w_{j-1,k-1} - w_{j+1,k-1} - w_{j-1,k+1}) \}
\end{aligned}$$

This equation can be rearranged as follows

$$A' v_{j,k-1} + B' v_{j,k} + C' v_{j,k+1} = D'$$

Similarly, the equation for w radial velocity at new station can be written and rearranged as follows:

$$A'' w_{j,k-1} + B'' w_{j,k} + C'' w_{j,k+1} = D''$$

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#### A.3.1.3 Correction for u,v,w

Static pressure variation is assumed in the streamwise direction for the parabolic nature of governing equations. Generally, the continuity equation is



not satisfied with the assumed static pressure variation. The static pressure is re-estimated so that the velocity components satisfy the continuity equation. The standard Poisson equation is solved for the cross section at given stage. The peculiar point in correcting process is that the mass conservation should be considered between periodic boundary surfaces.

The finite difference form of correction part of static pressure is as follows

$$P_{j,k}^* = \left\{ \frac{1}{\rho(\Delta y)^2} + \frac{1}{\rho(\Delta r)^2} \right\} \left\{ \frac{1}{\rho(\Delta y)^2} (P'_{j-1,k} - P'_{j+1,k}) + \frac{1}{\rho(\Delta r)^2} (P'_{j,k+1} + P'_{j,k-1}) \right. \\ \left. - \frac{u_{j,k} - U_{j,k}}{\Delta x} - \frac{V_{j,k}^* - V_{j-1,k}^*}{\Delta y} - \frac{W_{j,k}^* - W_{j,k-1}^*}{\Delta r} \right\}$$

and  $P_{j,k} = P_{j,k} + P_{j,k}^*$

The equations for correction of velocity components are as follows

$$V_{j,k}^* = - \frac{1}{\rho} \frac{1}{\Delta y} (P_{j+1,k}^* - P_{j,k}^*)$$

$$W_{j,k}^* = - \frac{1}{\rho} \frac{1}{\Delta r} (P_{j,k+1}^* - P_{j,k}^*)$$

$$u_{j,k}^* = - \frac{1}{\rho} \left( \frac{\partial P}{\partial x} \right)_{j,k}^*$$

and

$$\left( \frac{\partial P}{\partial x} \right)_{j,k}^* = \frac{\frac{\dot{m}}{\rho} - \sum u_{j,k} \Delta y \Delta r}{\sum - \frac{1}{\rho} \Delta y \Delta r}$$

where  $\dot{m}$  is the flow rate between periodic boundary surfaces.

### A.3.2 y-implicit step

The similar equations to those in 3.1 can be easily developed for y-implicit step. At this step of calculation, the periodic boundary condition is applied and this step is much like correction step, whereas the r-implicit step is much like predict step.

#### A.4 Calculation of k and $\epsilon$ for Subsequent Step

The effective kinetic eddy viscosity is calculated from the equation

$$\nu_t = C_\mu \frac{k^2}{\epsilon}$$

Therefore, at each step k and  $\epsilon$  should be calculated from the transport equations of k and  $\epsilon$ .

The finite difference forms of these transport equations is straightforward. The equations can be rearranged like

$$Ak_{j,k-1} + Bk_{j,k} + Ck_{j,k+1} = D$$

#### A.5 Further Correction for Calculation

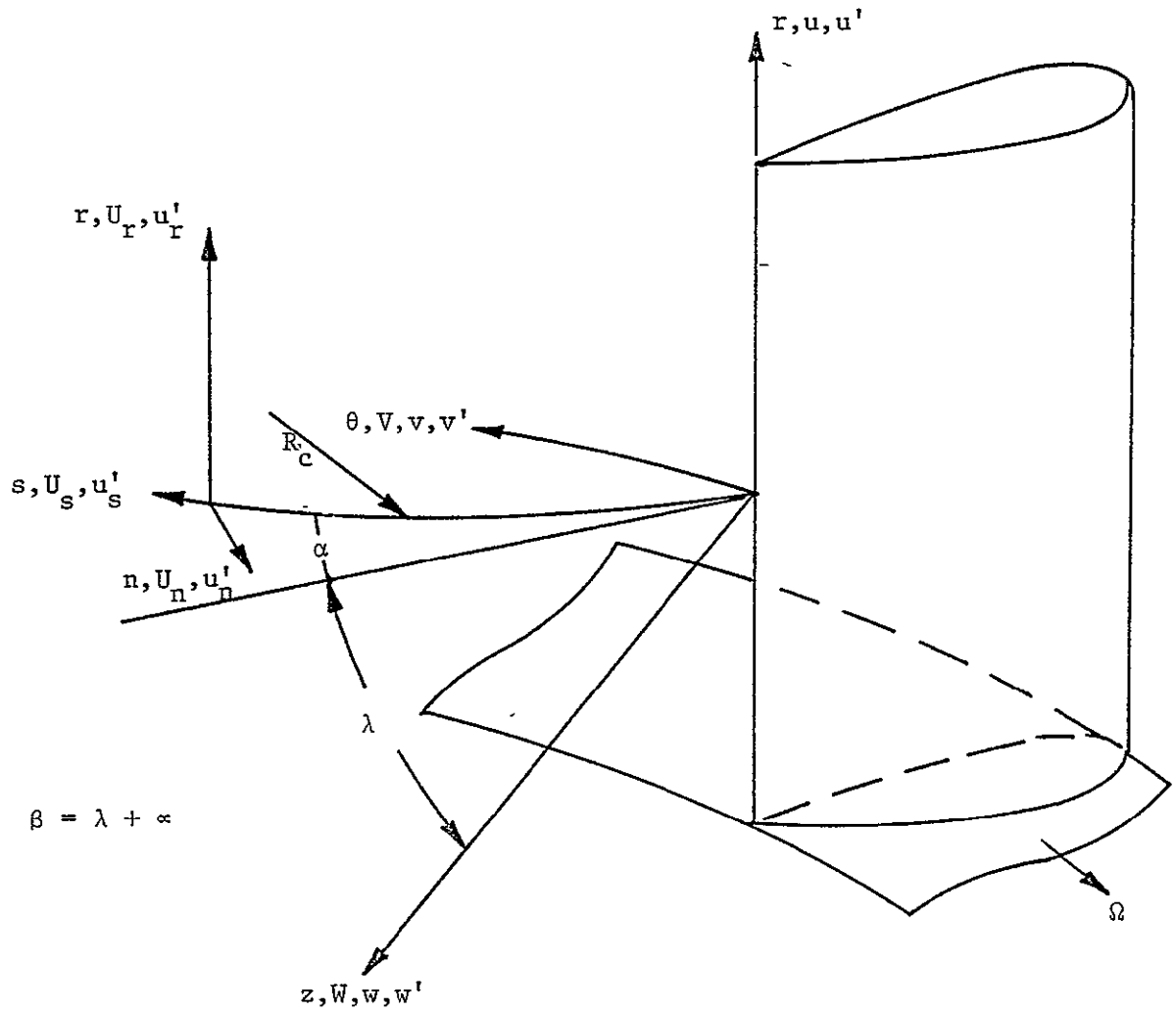
The above sections briefly explain the numerical scheme to be used for the calculation of rotor wake. The turbulence model includes the effects of rotation and streamline curvature. The static pressure distribution will be stored and calculated three dimensionally. The result of calculation will be compared closely with experimental data. The possibility of correction on turbulence model as well as on numerical scheme will be studied.

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Figure 1: Coordinate system and notations used

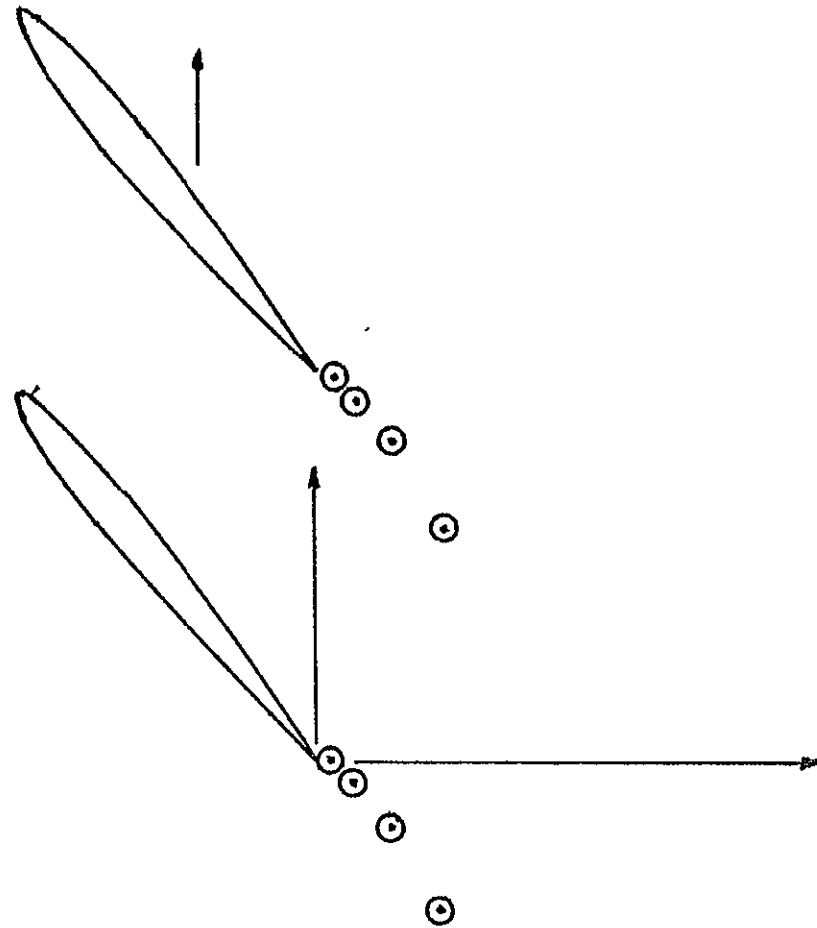


Figure 2 : Curvature of Rotor Wake at  $r/r_t = 0.860$ ,  $i = 10^\circ$  from Rotating Triaxial Probe Data.

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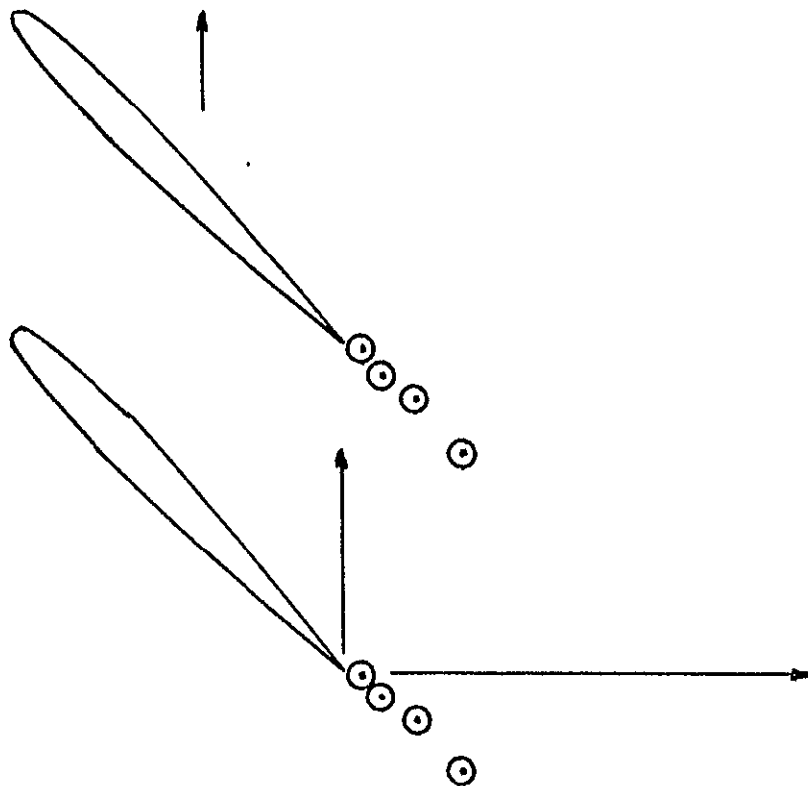


Figure 3 : Curvature of Rotor Wake at  $r/r_t = 0.721$ ,  $i = 10^\circ$  from Rotating Triaxial Probe Data.

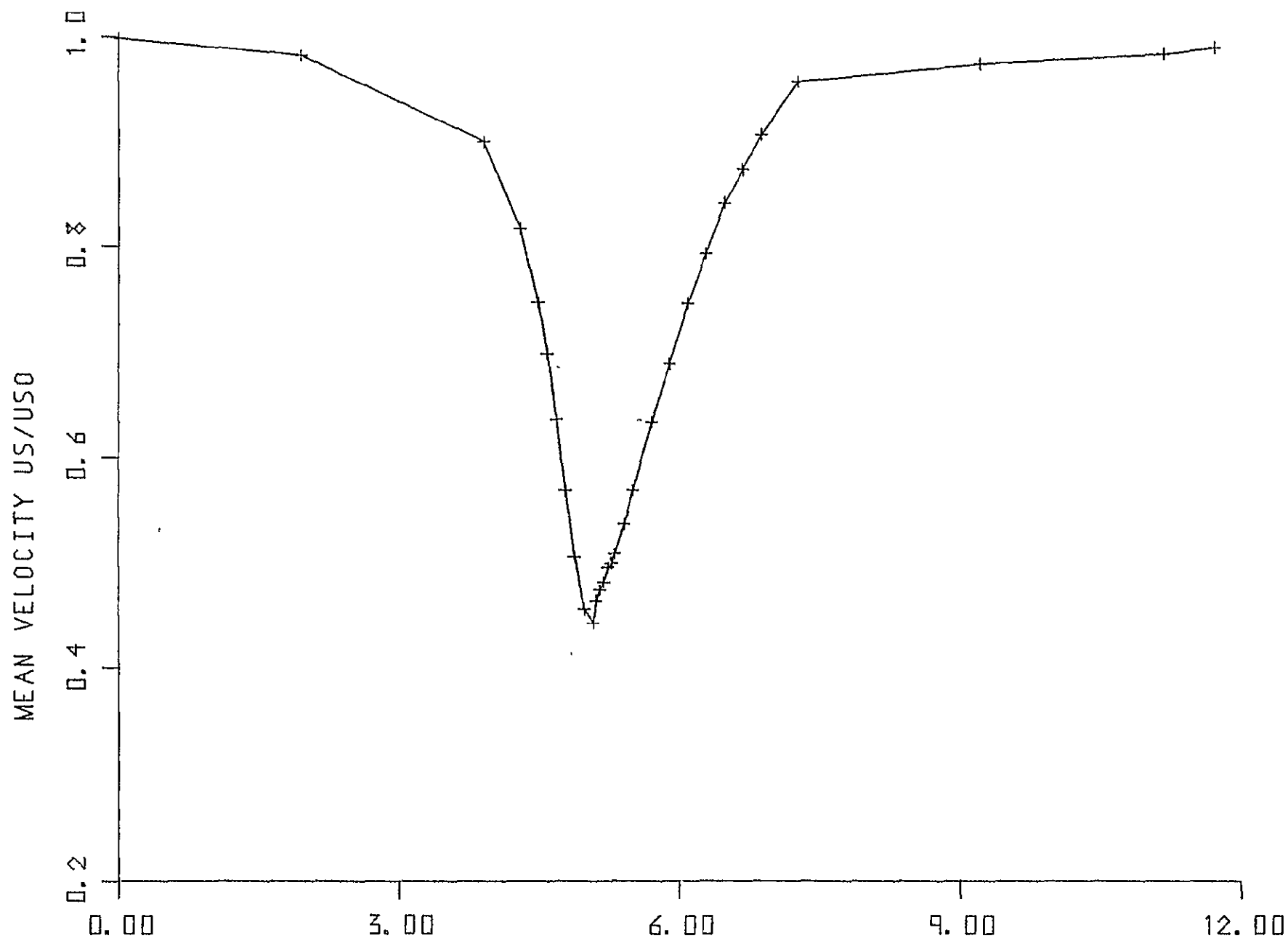


Figure 4:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)



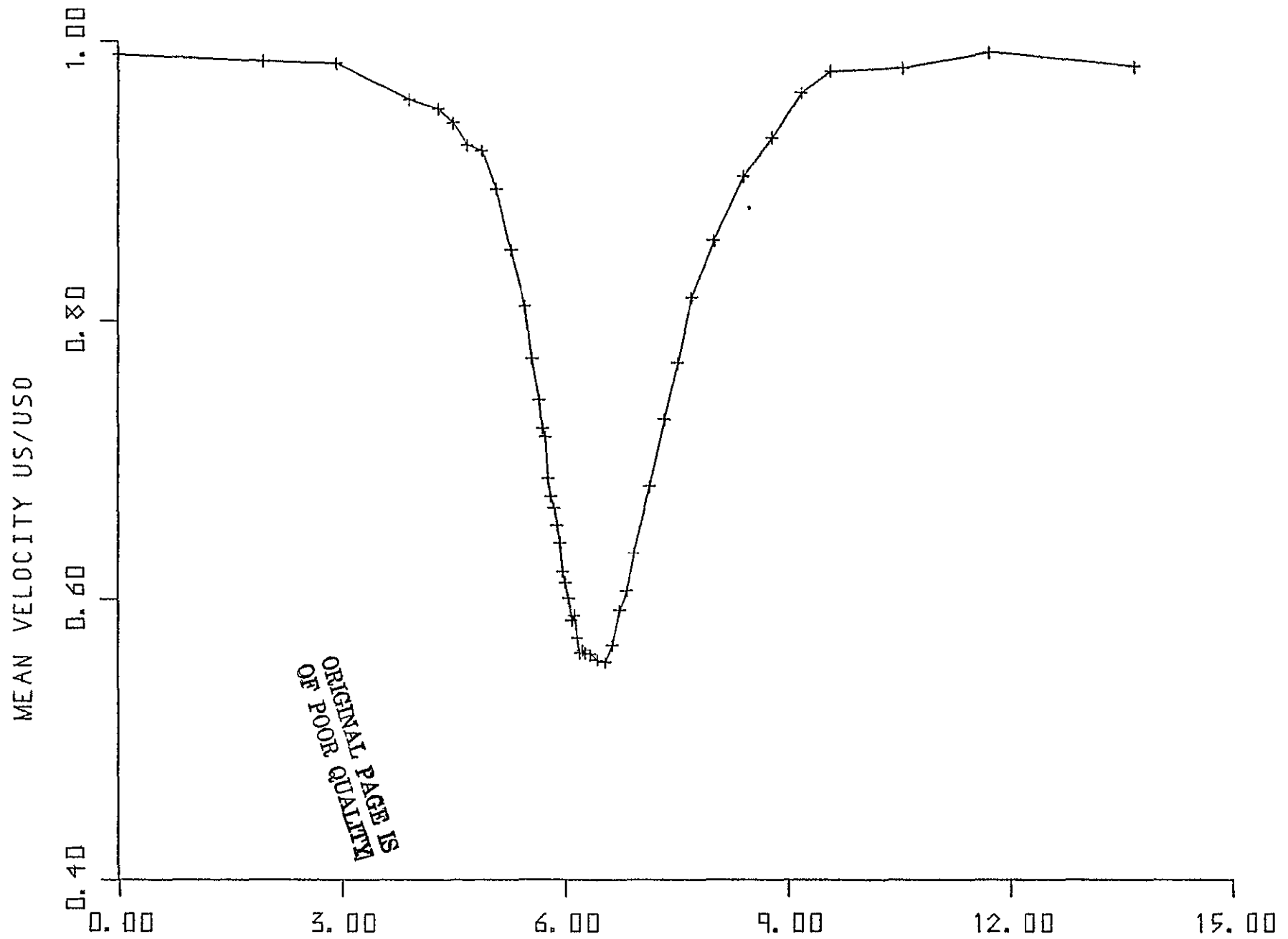


Figure 5:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\theta$ (DEG.)

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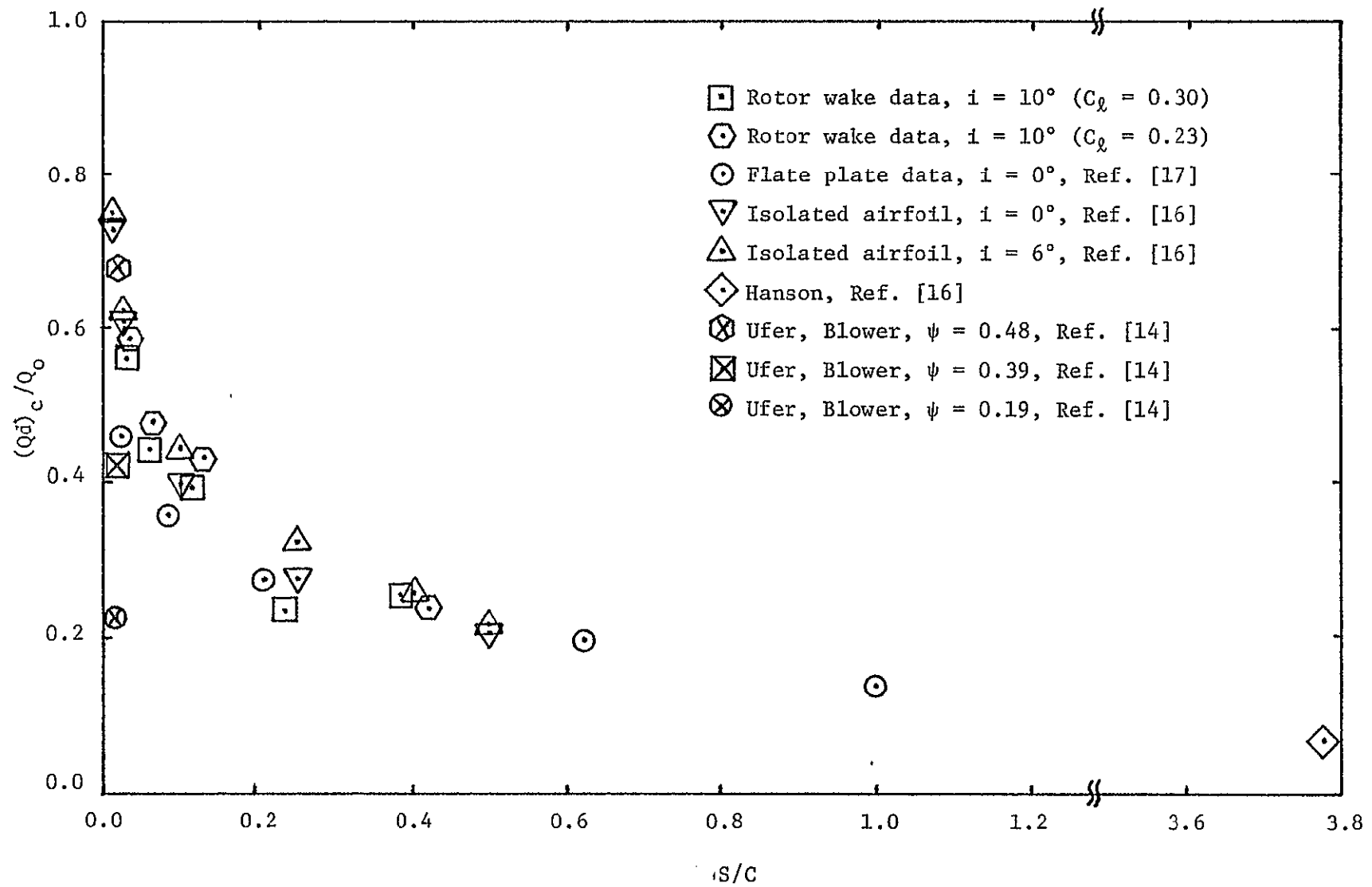
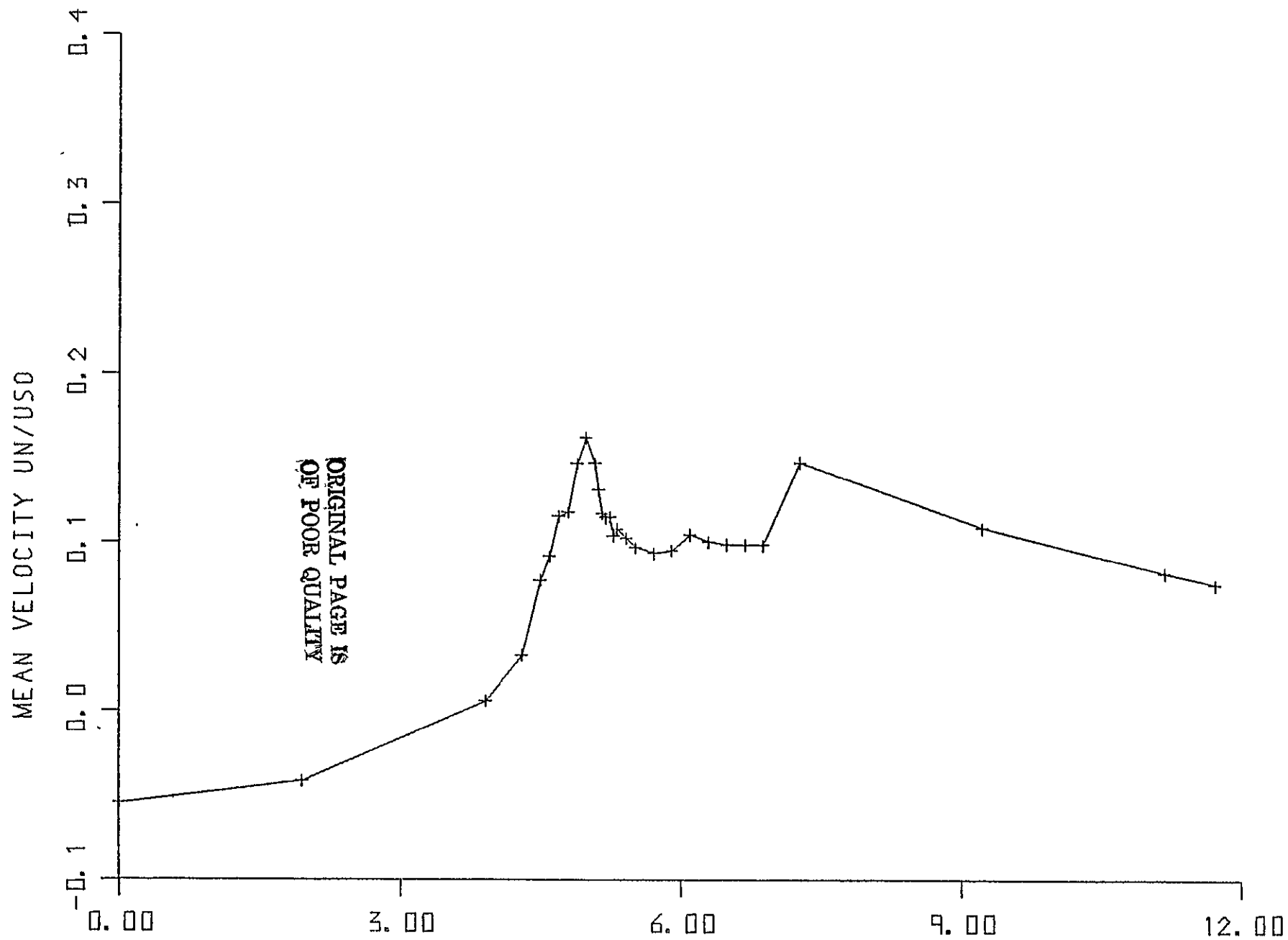


Figure 6: Decay of mean velocity at wake centerline



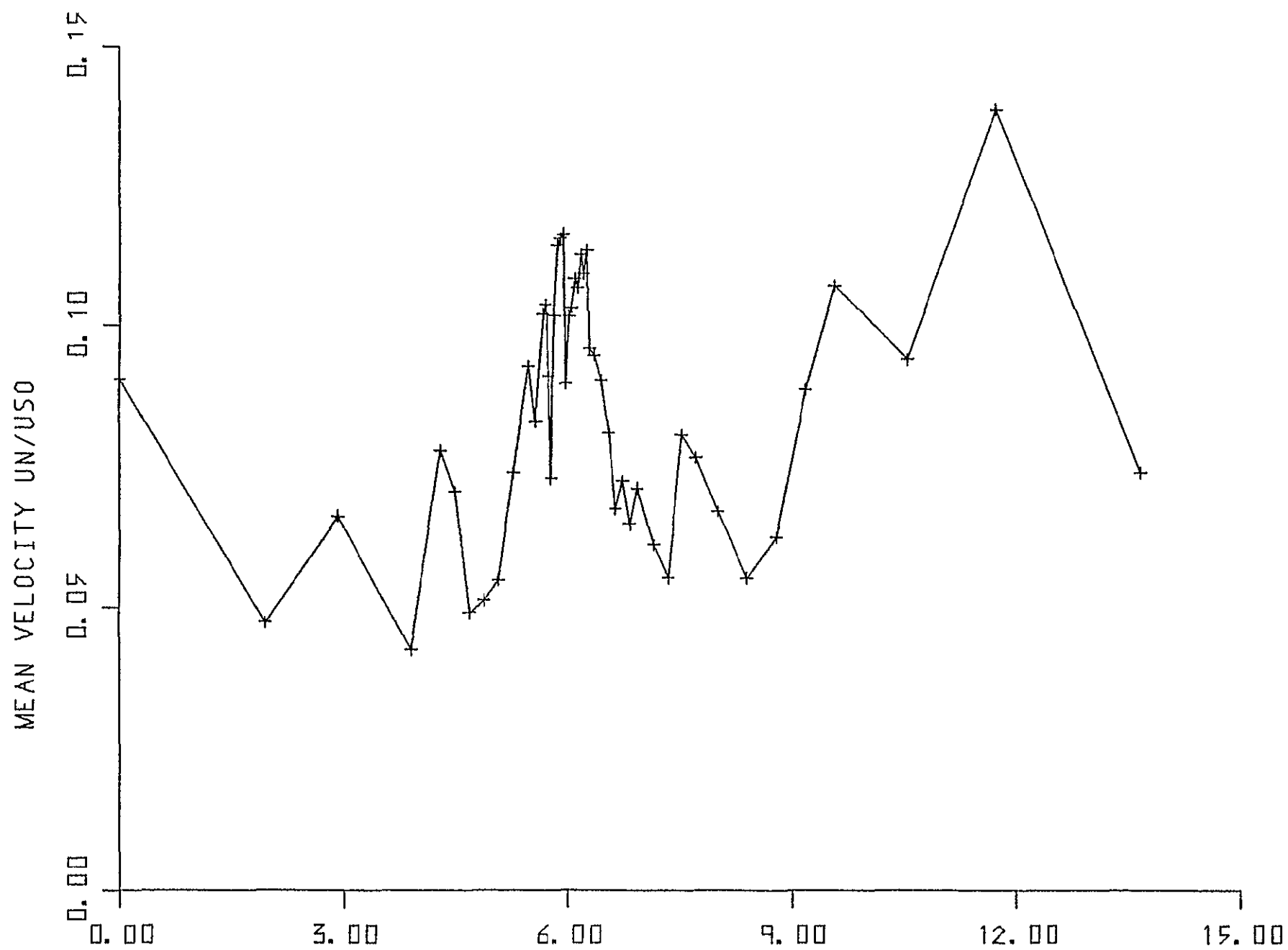


Figure 8:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\theta<DEG.$

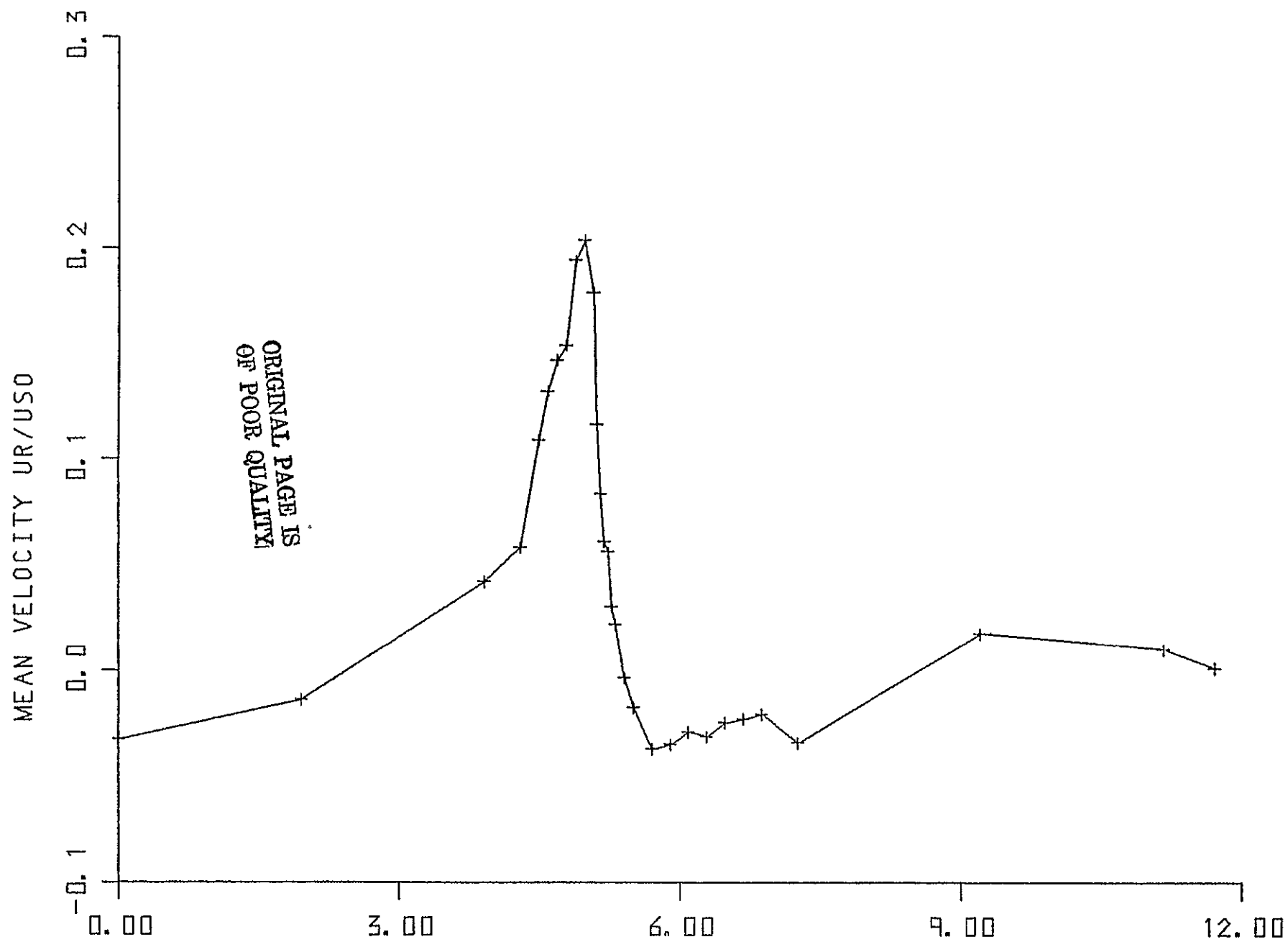


Figure 9:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)

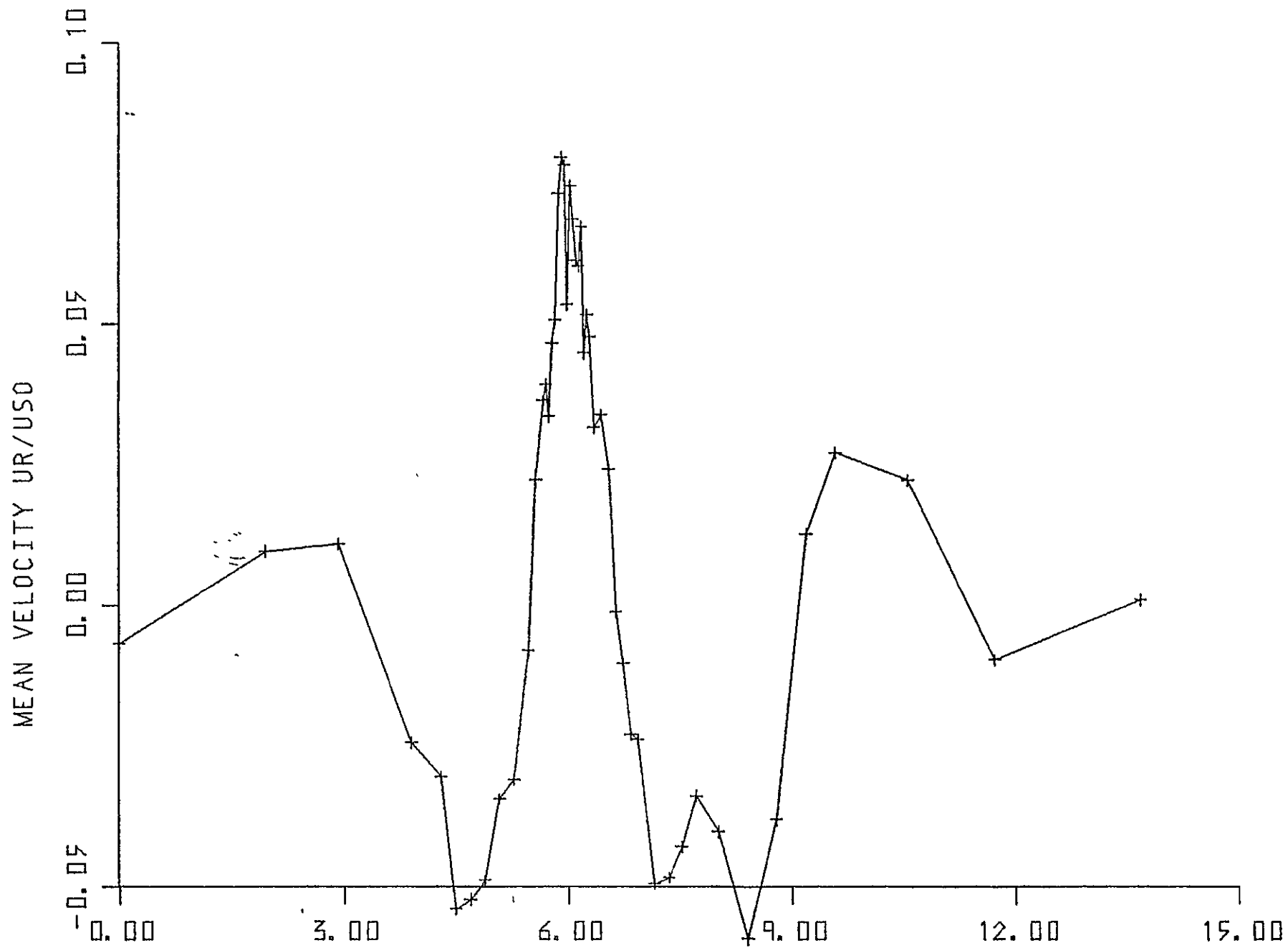


Figure 10:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\theta<DEG.$

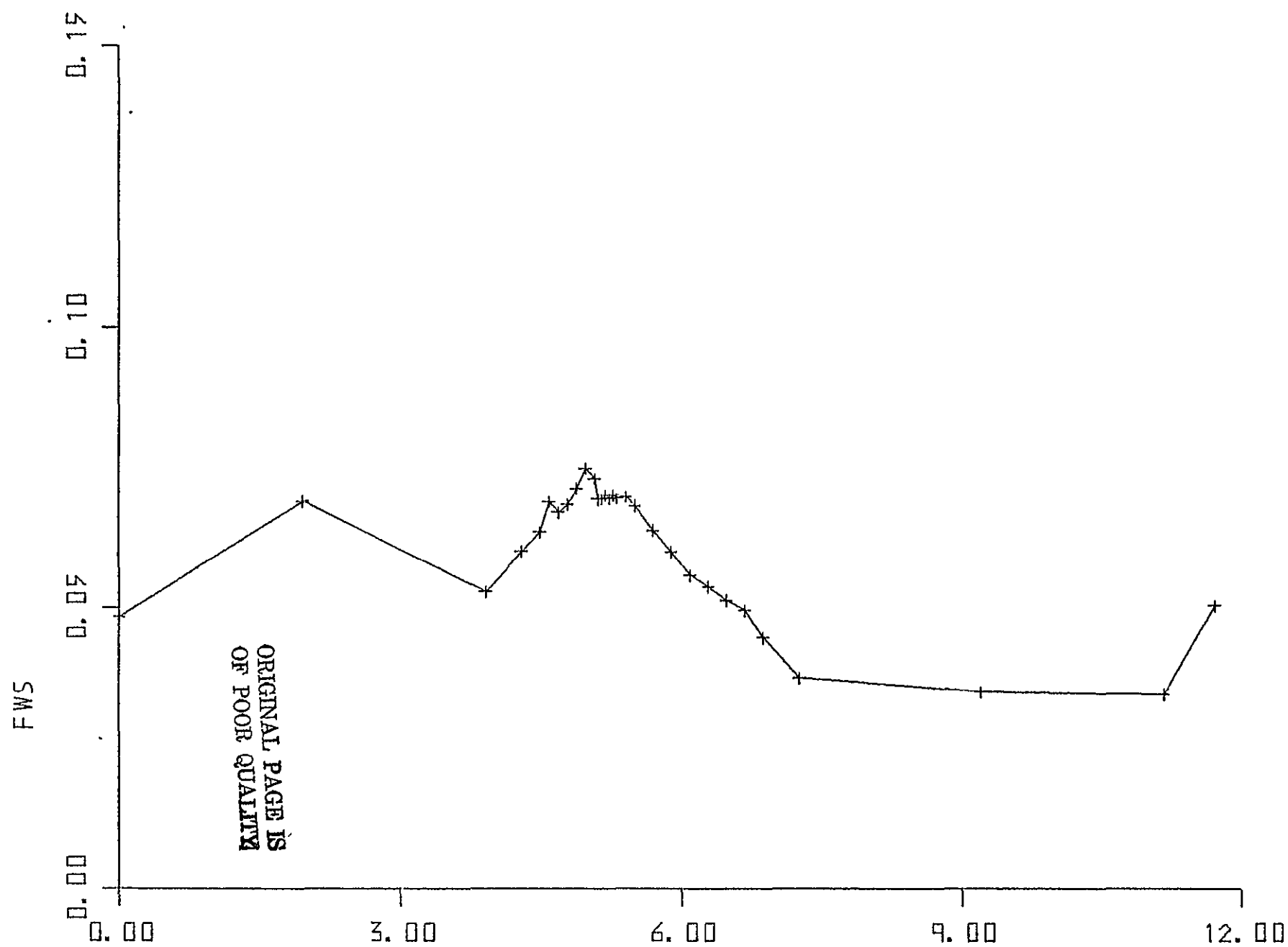


Figure 11:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)

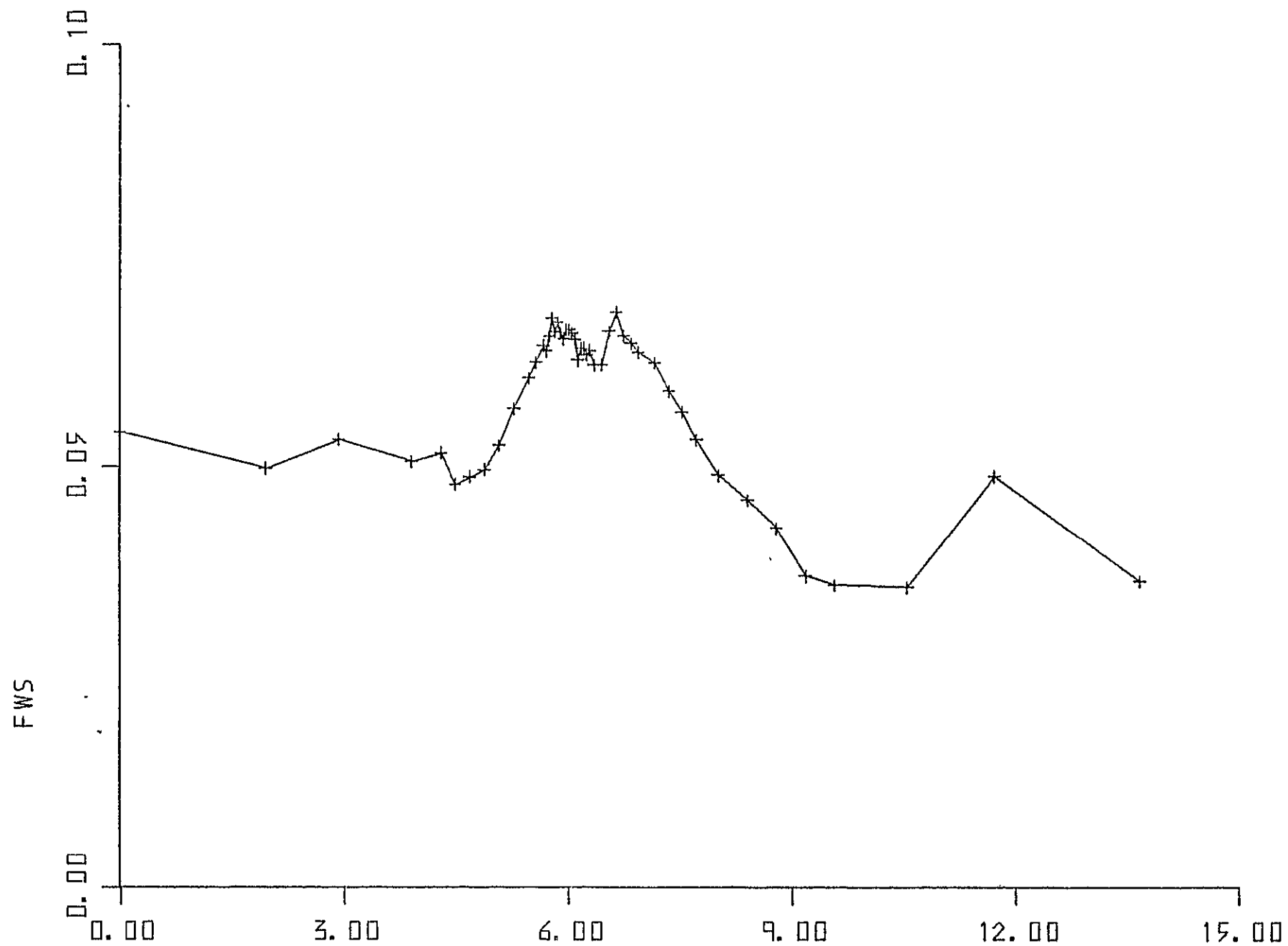


Figure 12:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ , THETA<DEG. >



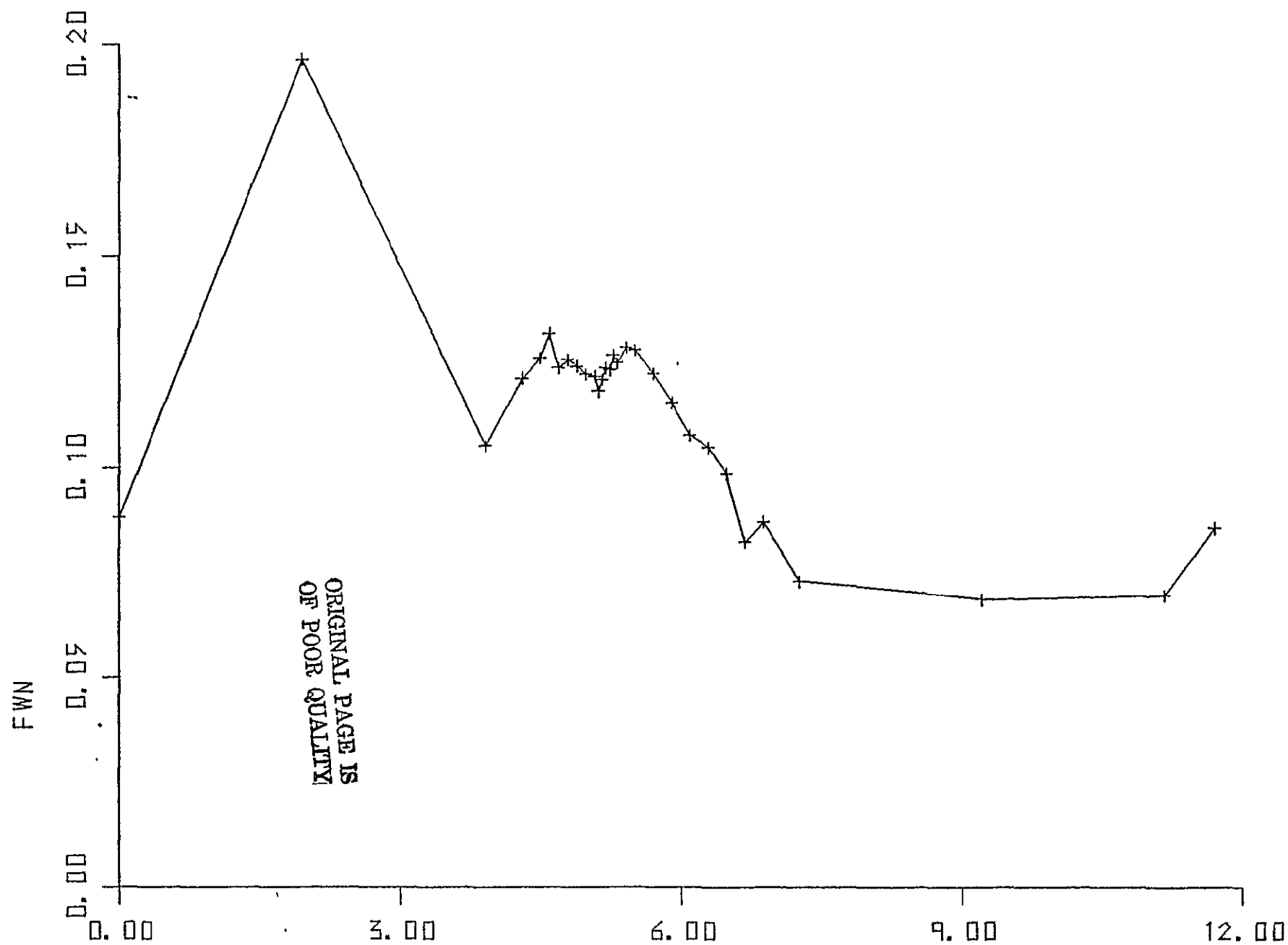


Figure 13:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)

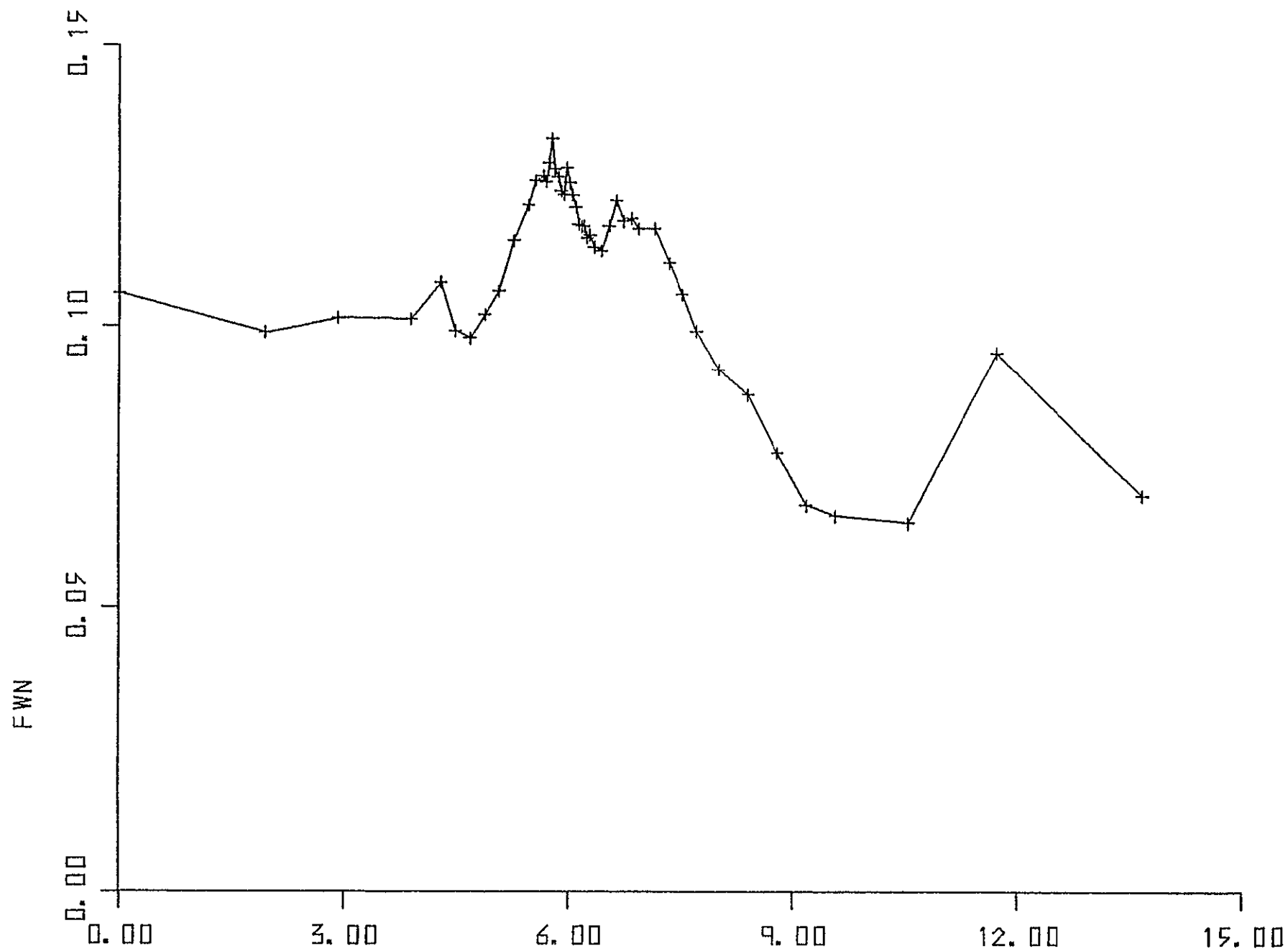


Figure 14:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ , THETA( DEG. )

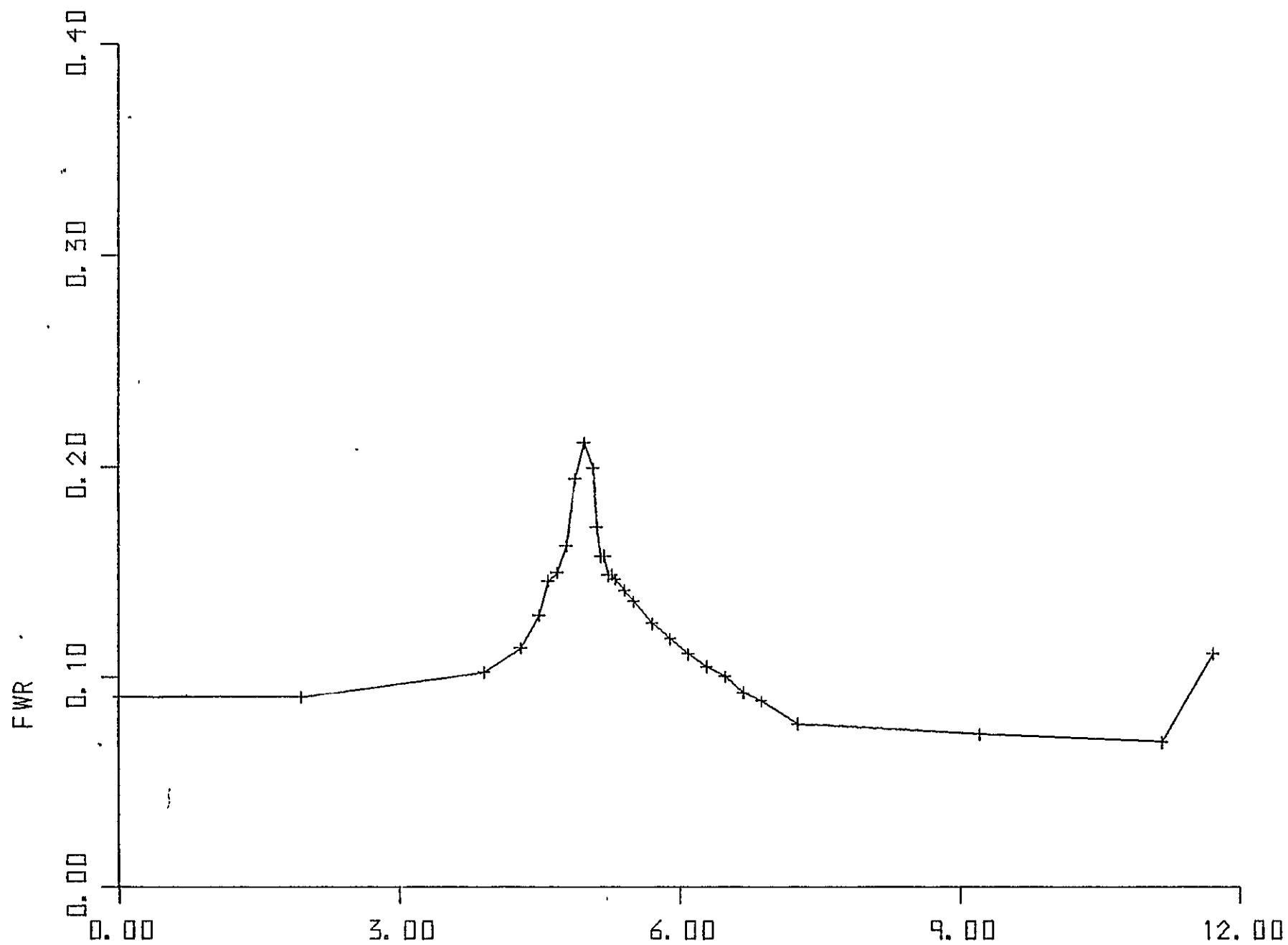


Figure 15:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)

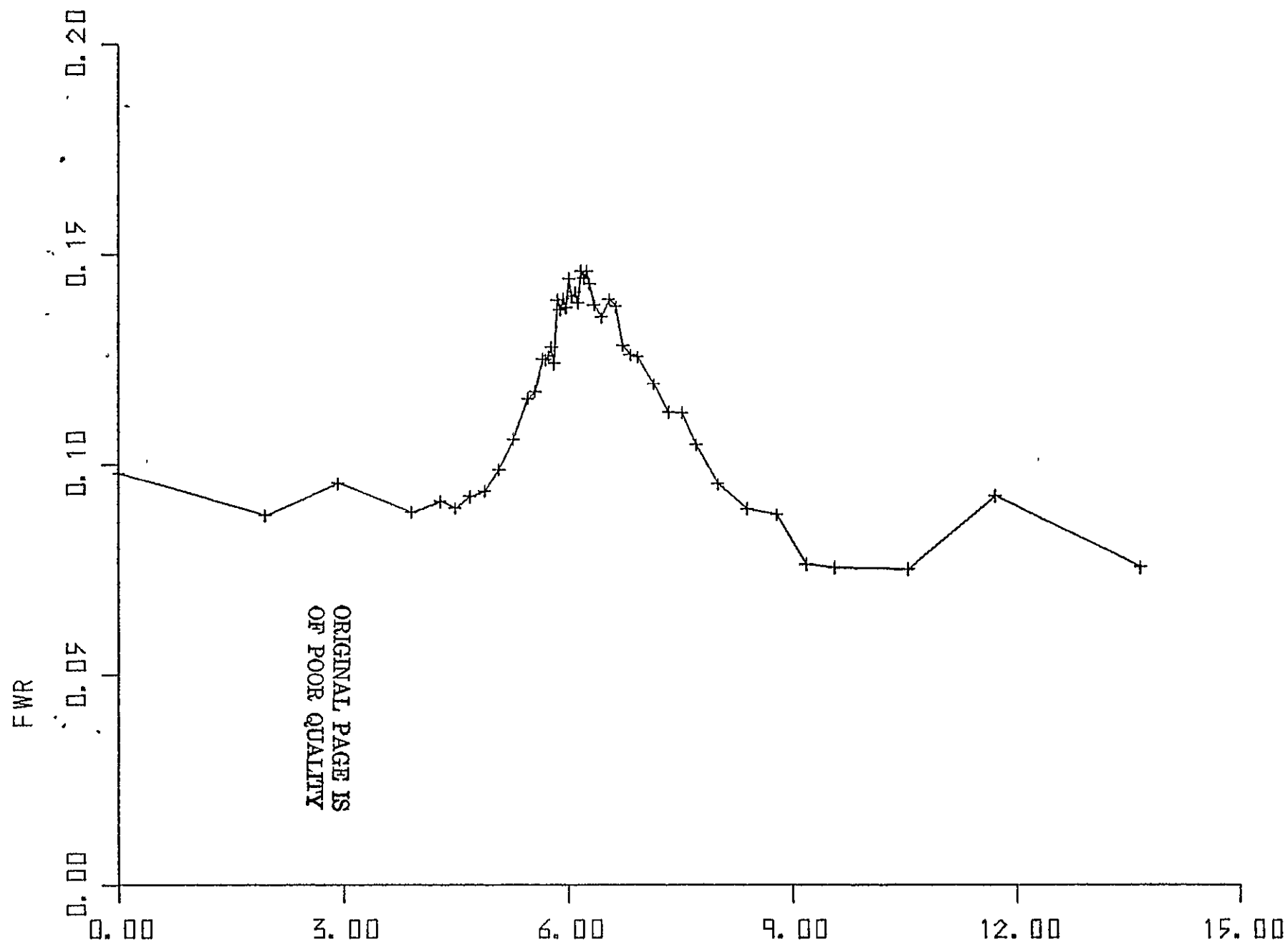


Figure 16:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\theta<\text{DEG.}$

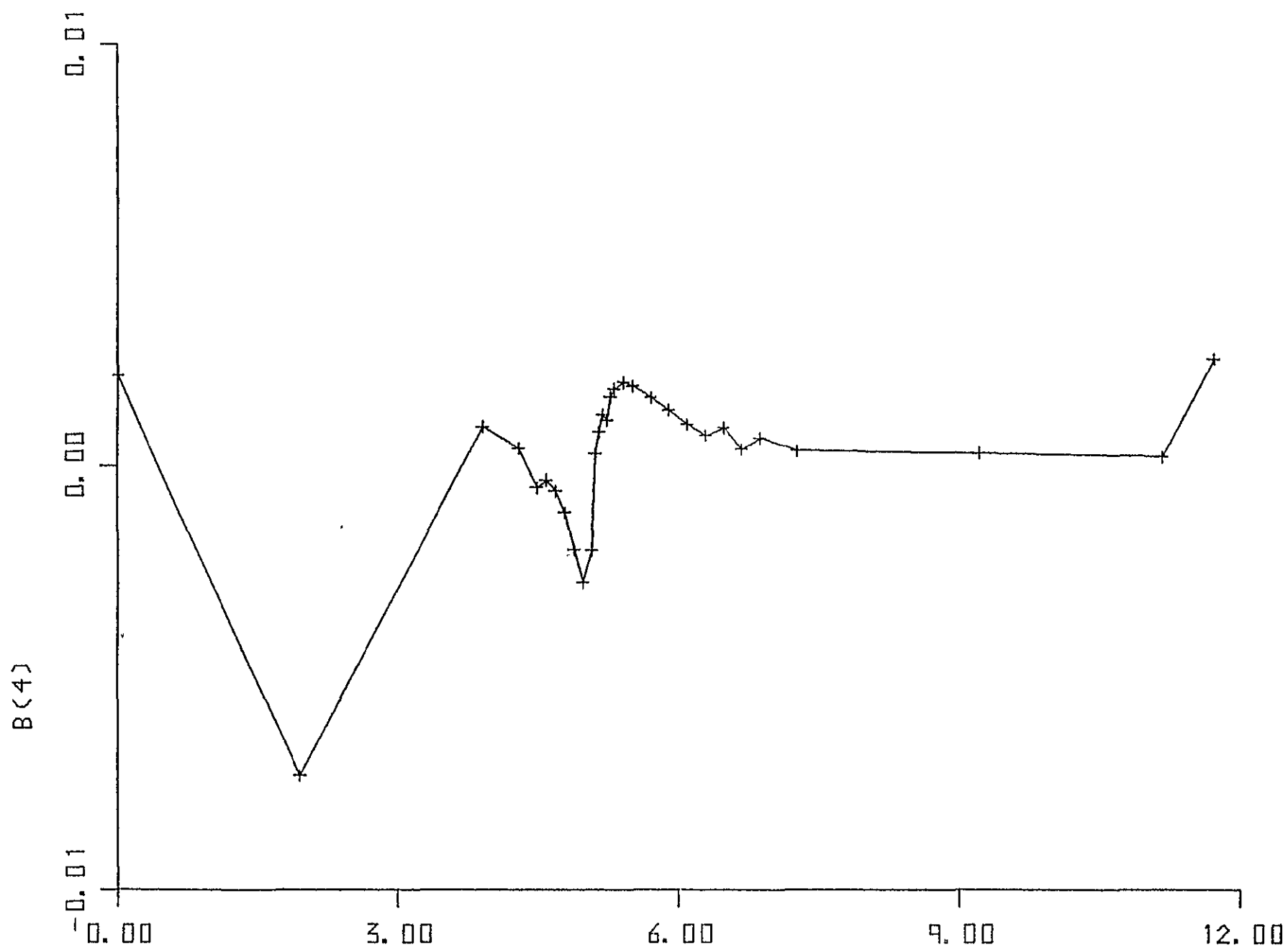


Figure 17:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\text{THETA (DEG.)}$

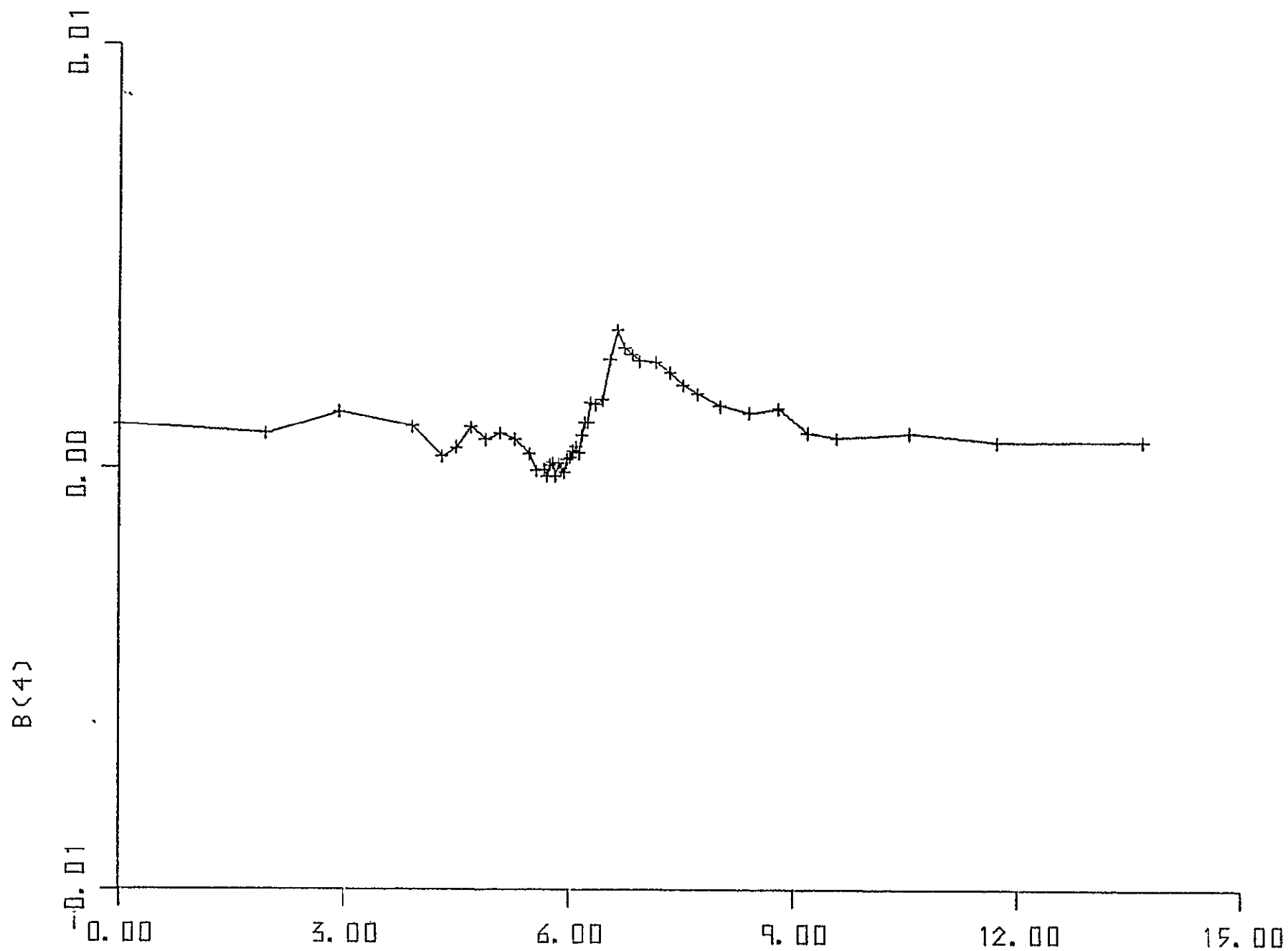


Figure 18:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\text{THETA (DEG.)}$

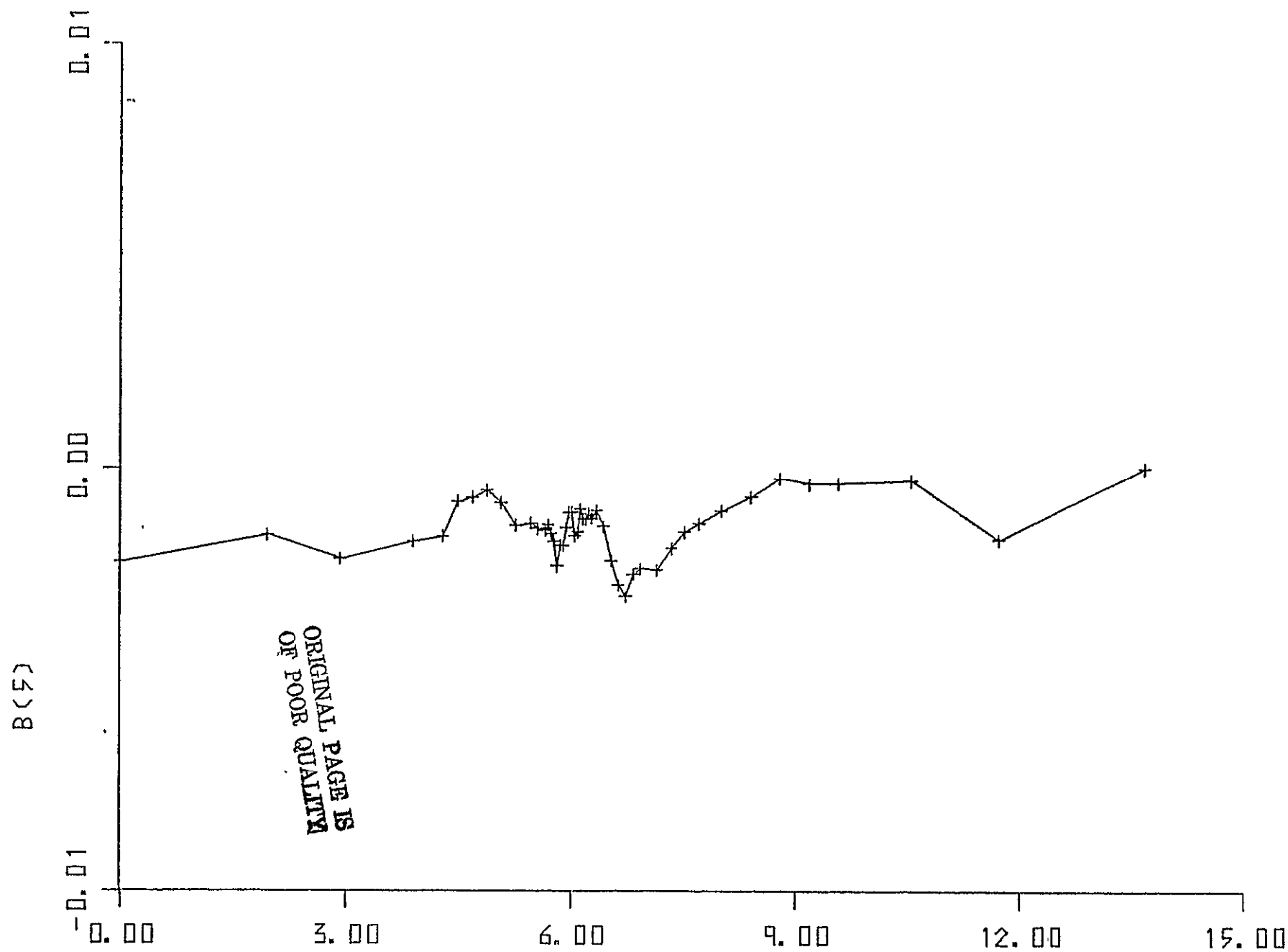


Figure 19:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\theta$ (DEG.)

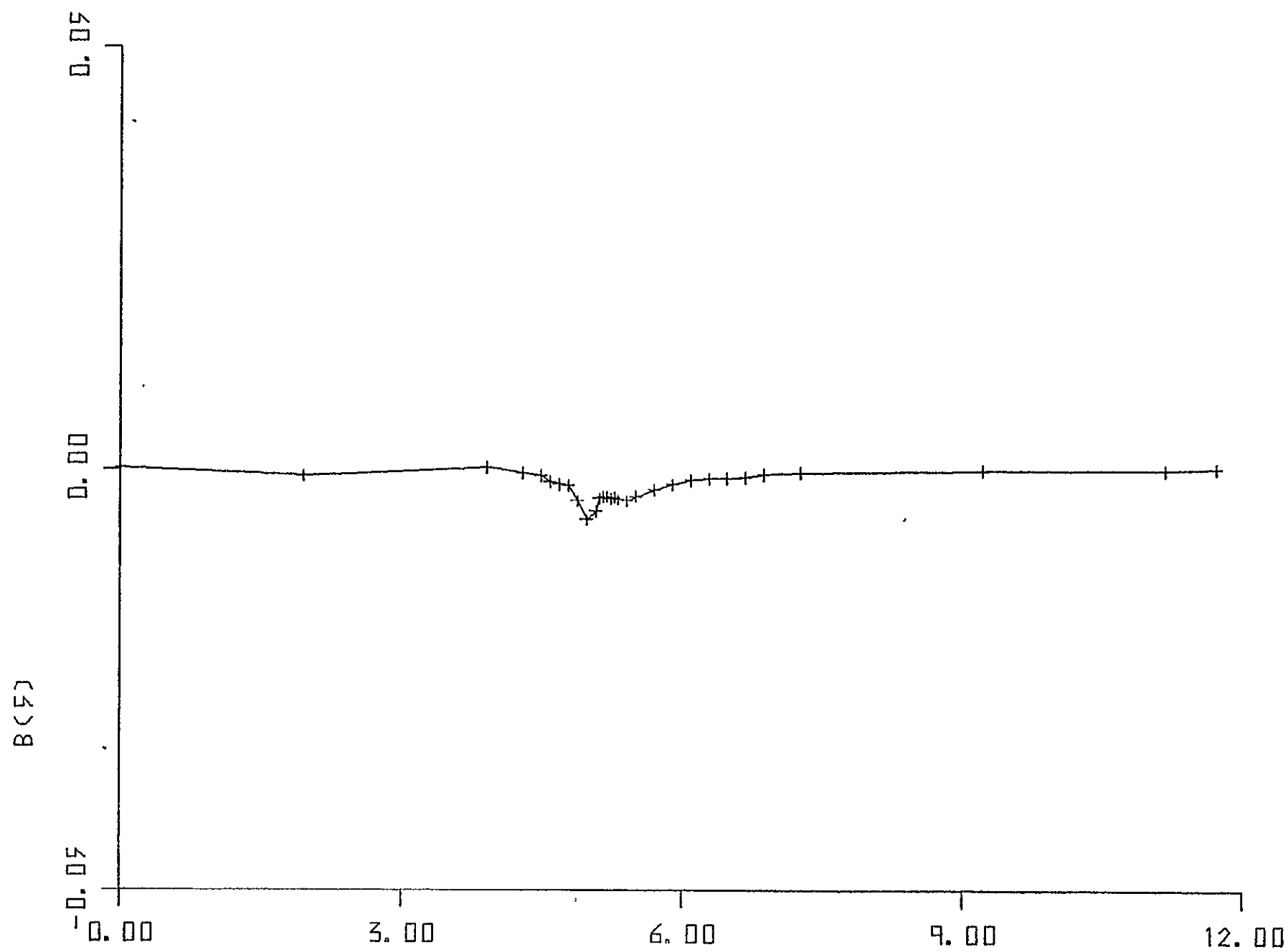


Figure 20:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)



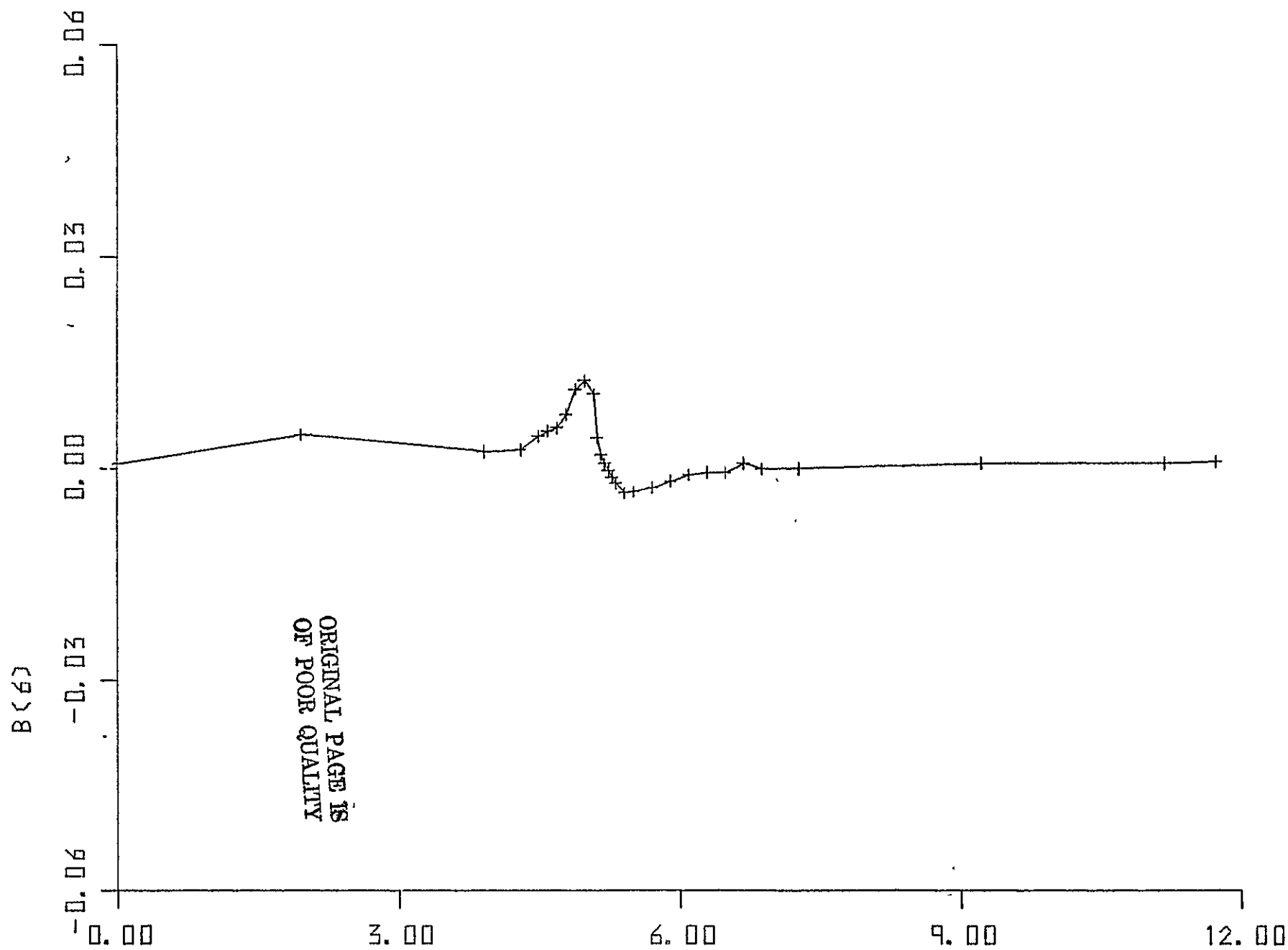


Figure 21:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\theta$  (DEG.)

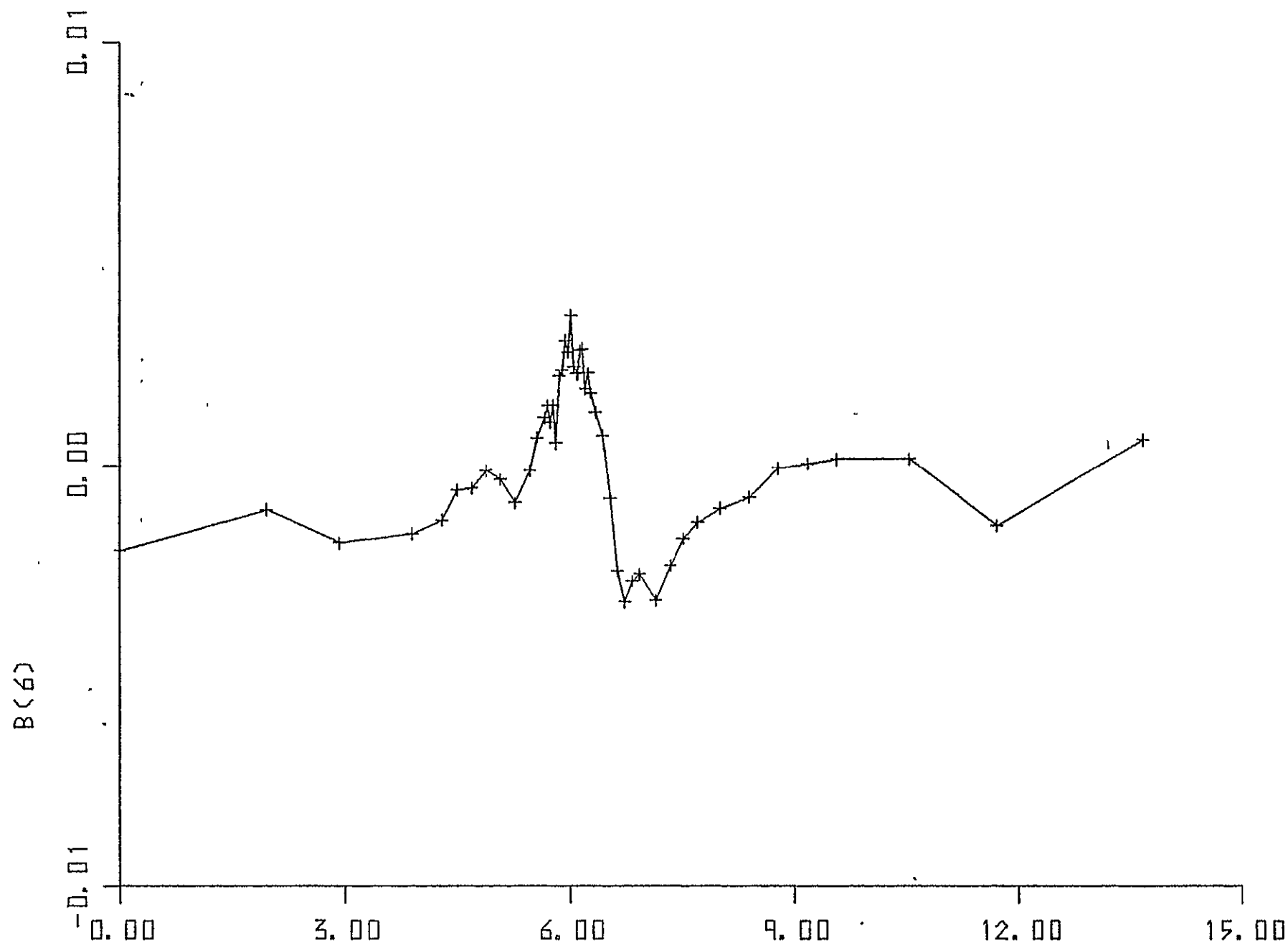


Figure 22:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\text{THETA (DEG.)}$

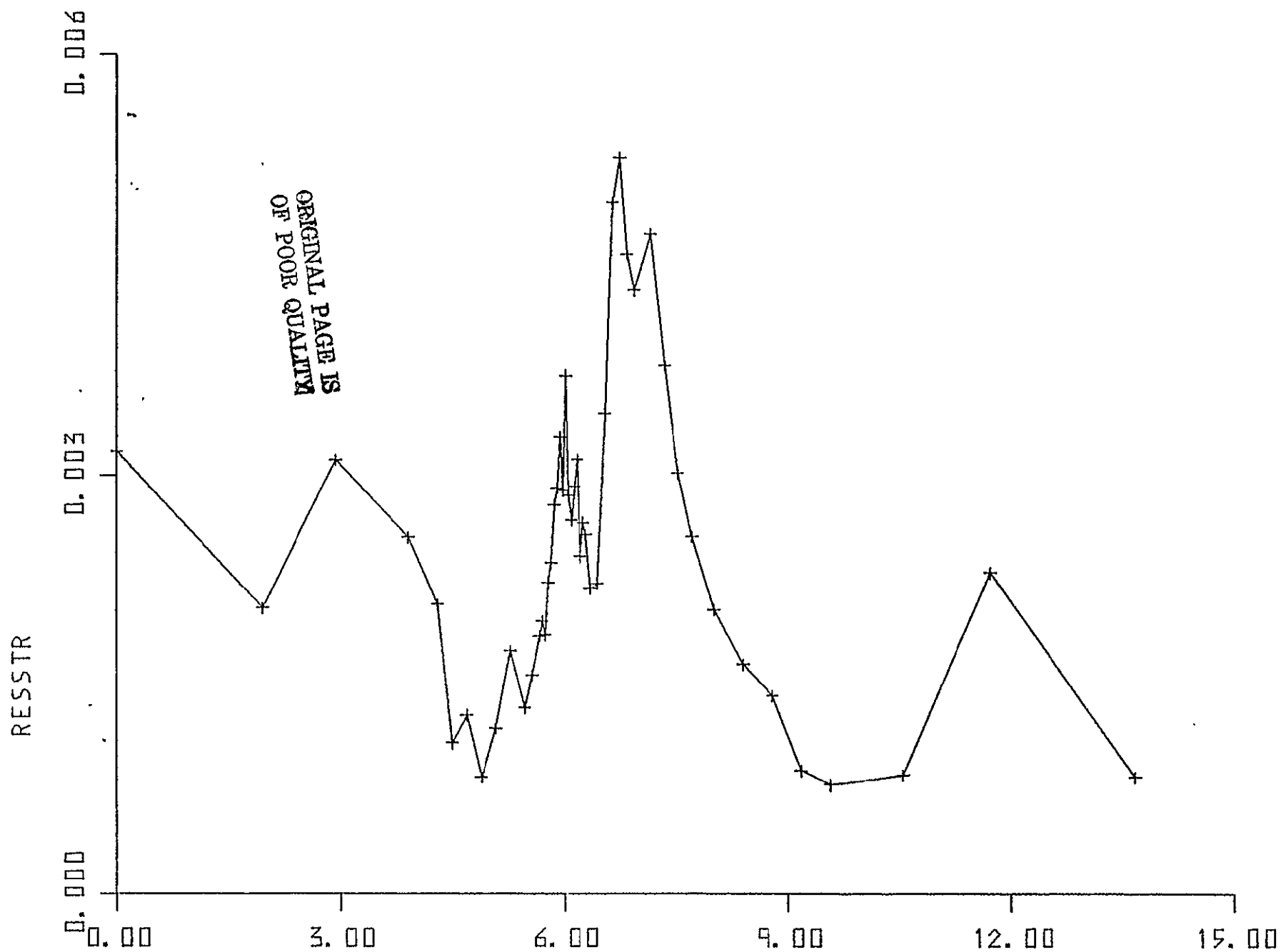


Figure 23:  $X/C=0.042$ ,  $R/RT=0.721$ ,  $I=10$ ,  $\text{THETA}(\text{DEG.})$

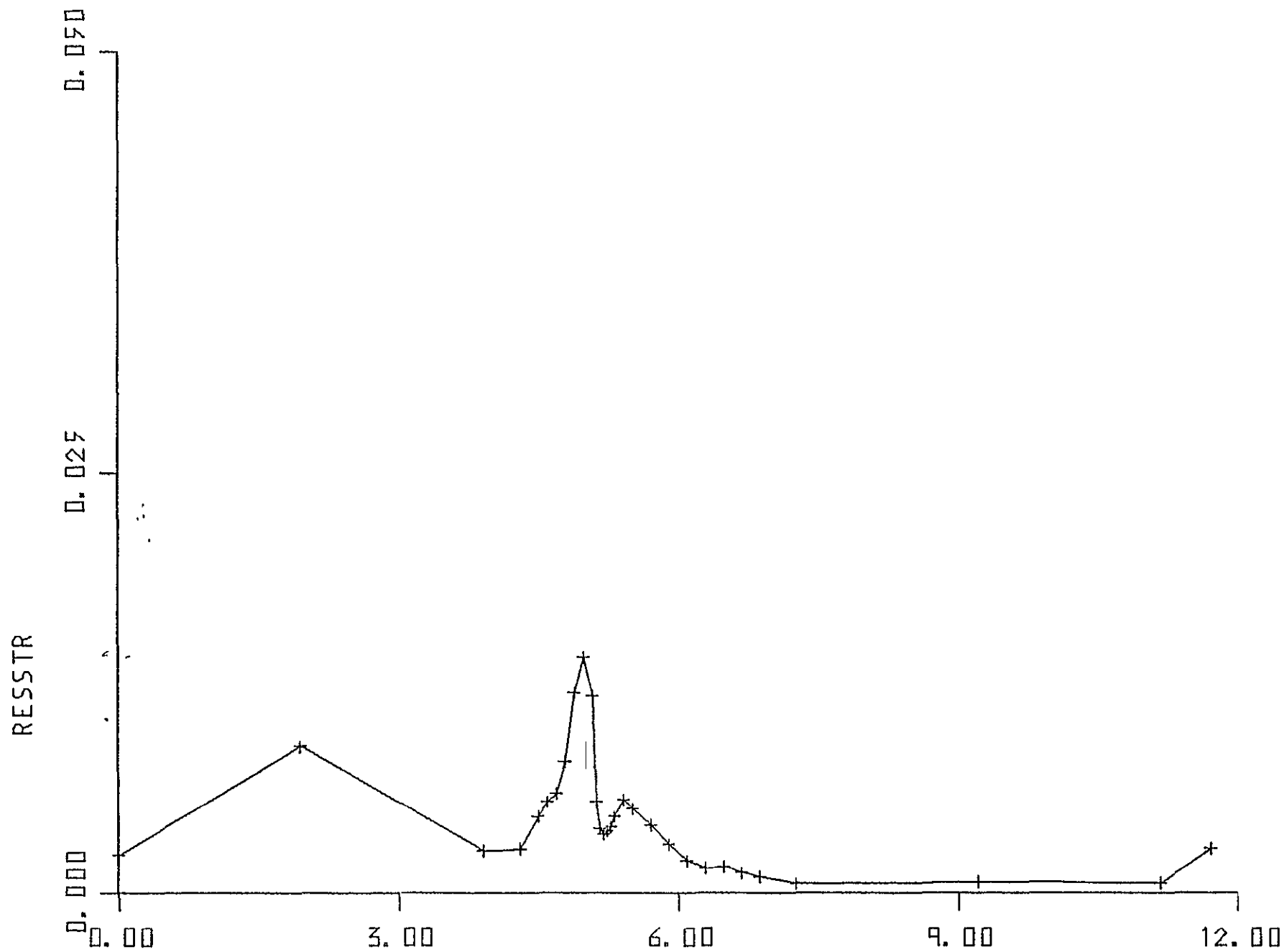
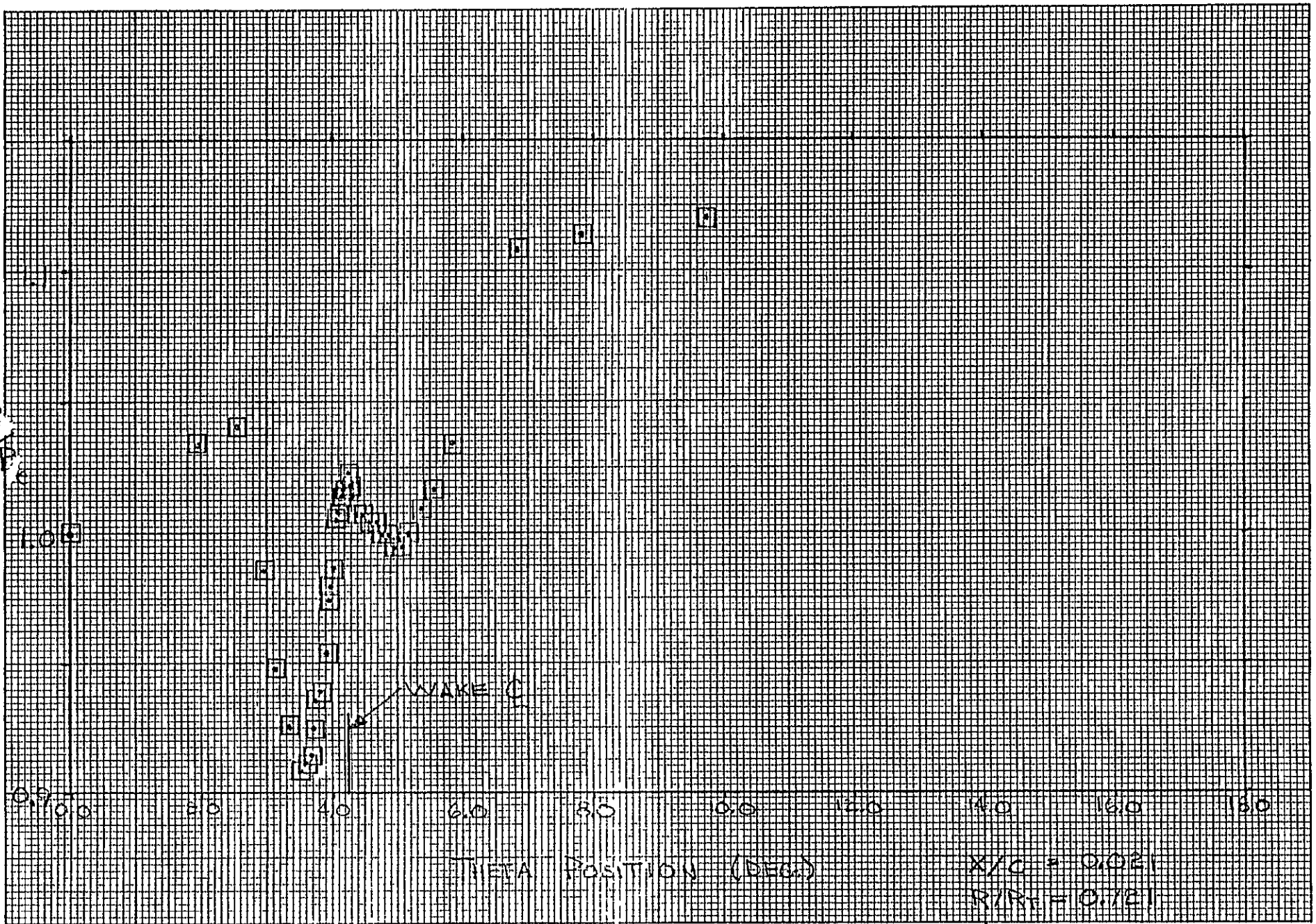


Figure 24:  $X/C=0.021$ ,  $R/RT=0.721$ ,  $I=10$ , THETA (DEG.)

Figure 25: Static pressure distribution across the wake ( $x/c = 0.021$ ,  $R/R_T = 0.721$ ,  $i = 10^\circ$ )

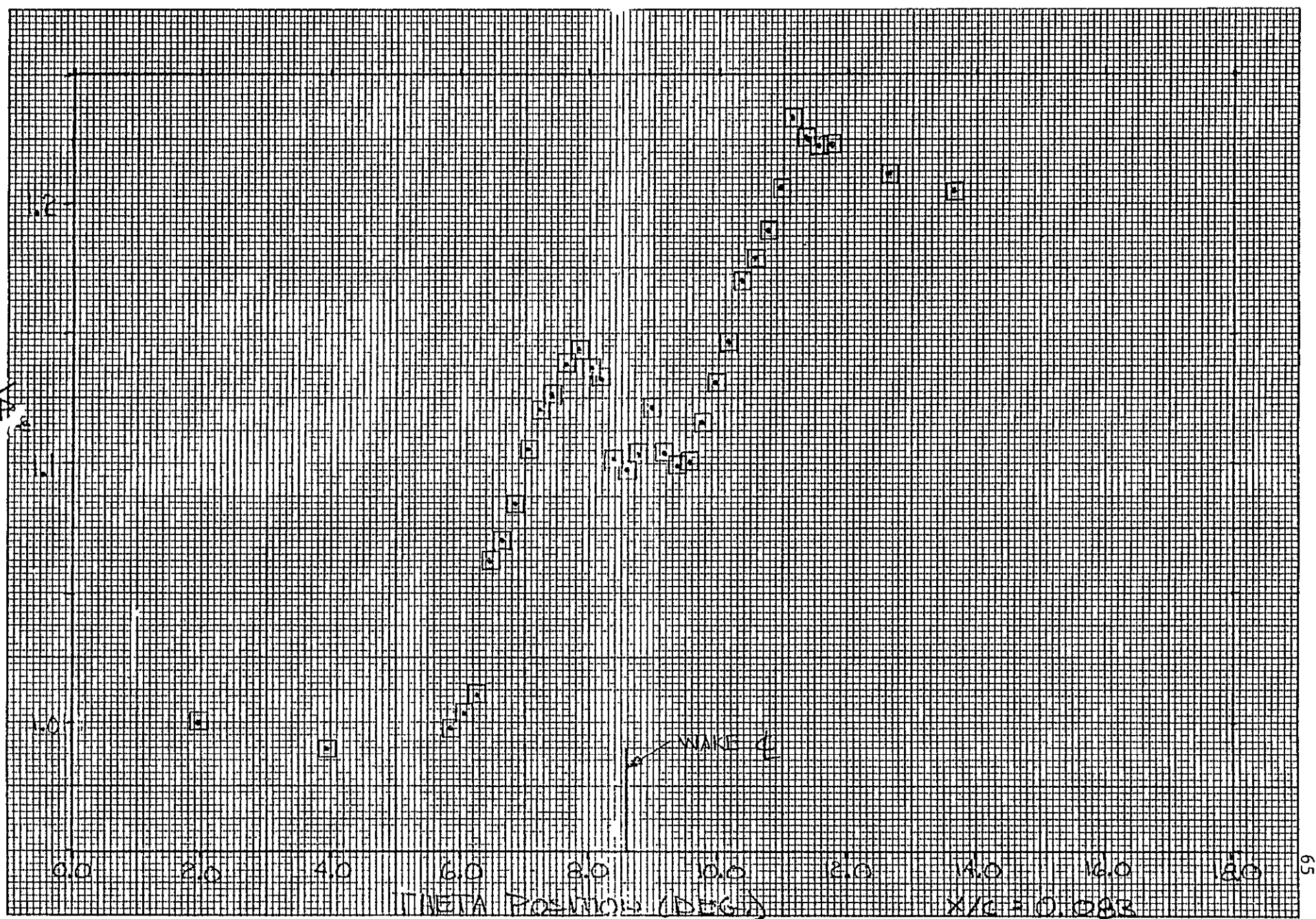


Figure 26: Static pressure distribution across the wake ( $x/c = 0.083$ ,  $R/R_T = 0.721$ ,  $i = 10^\circ$ )

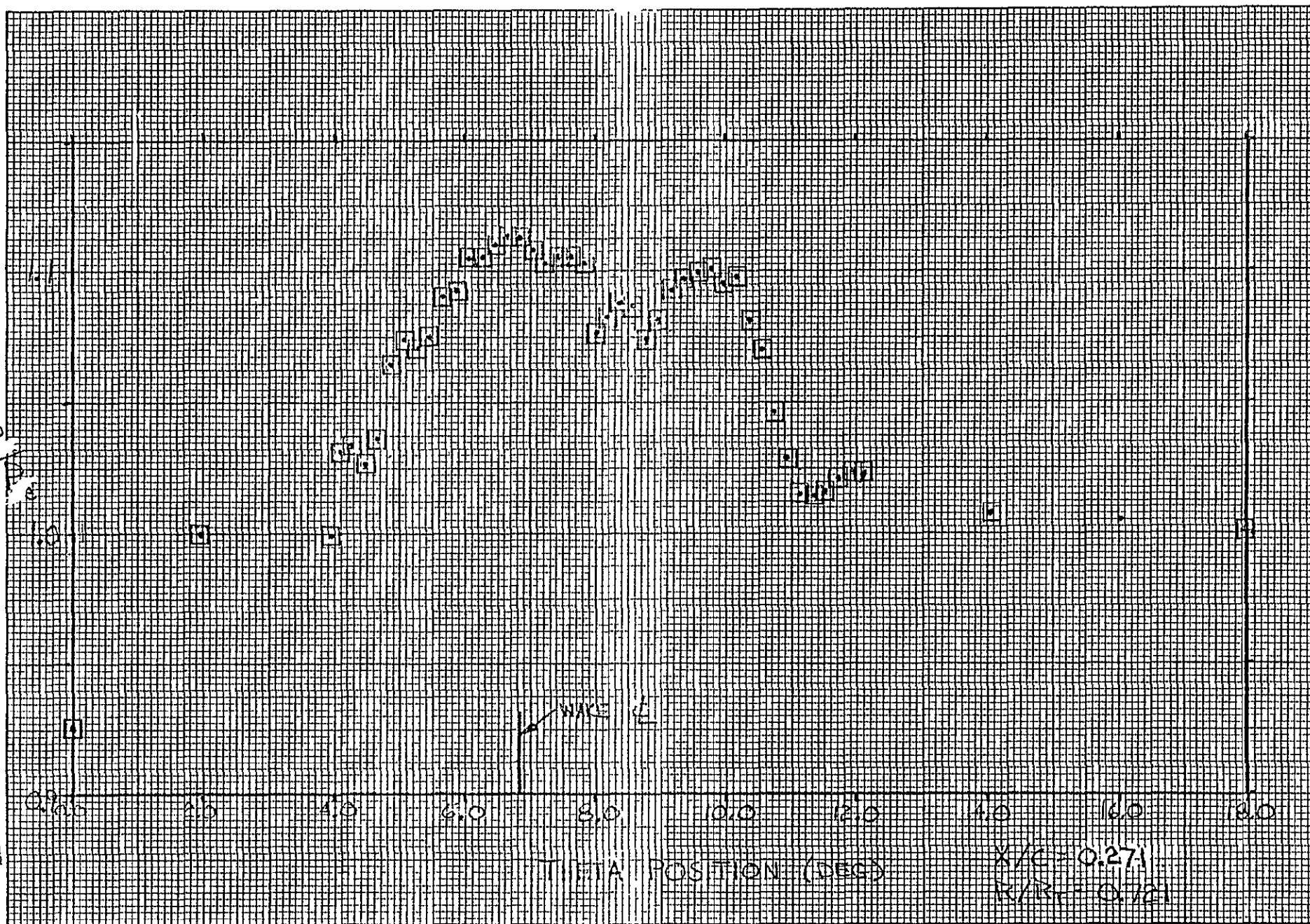


Figure 27: Static pressure distribution across the wake ( $x/c = 0.271$ ,  $R/R_T = 0.721$ ,  $i = 10^\circ$ )  $i = 10^\circ$



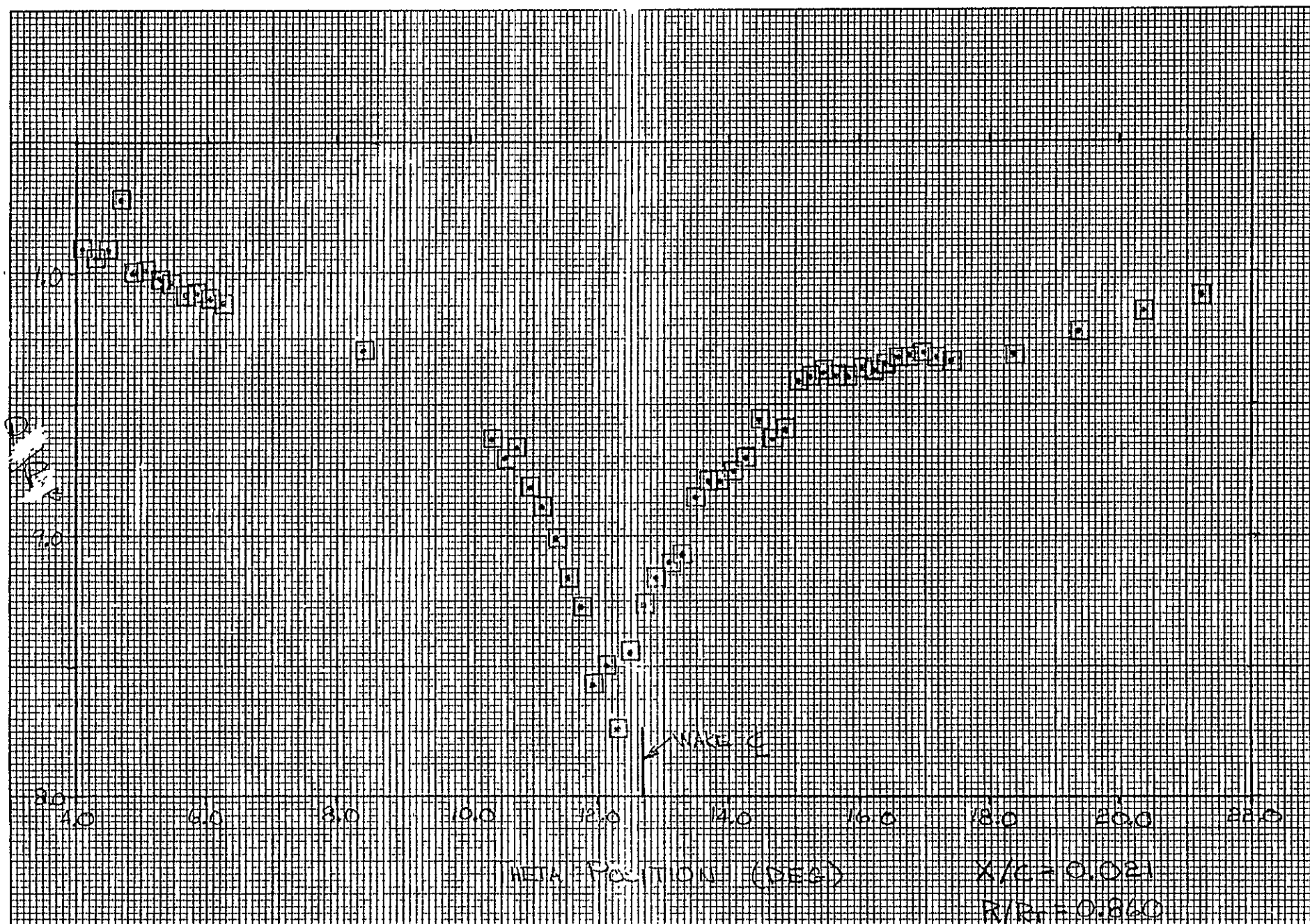
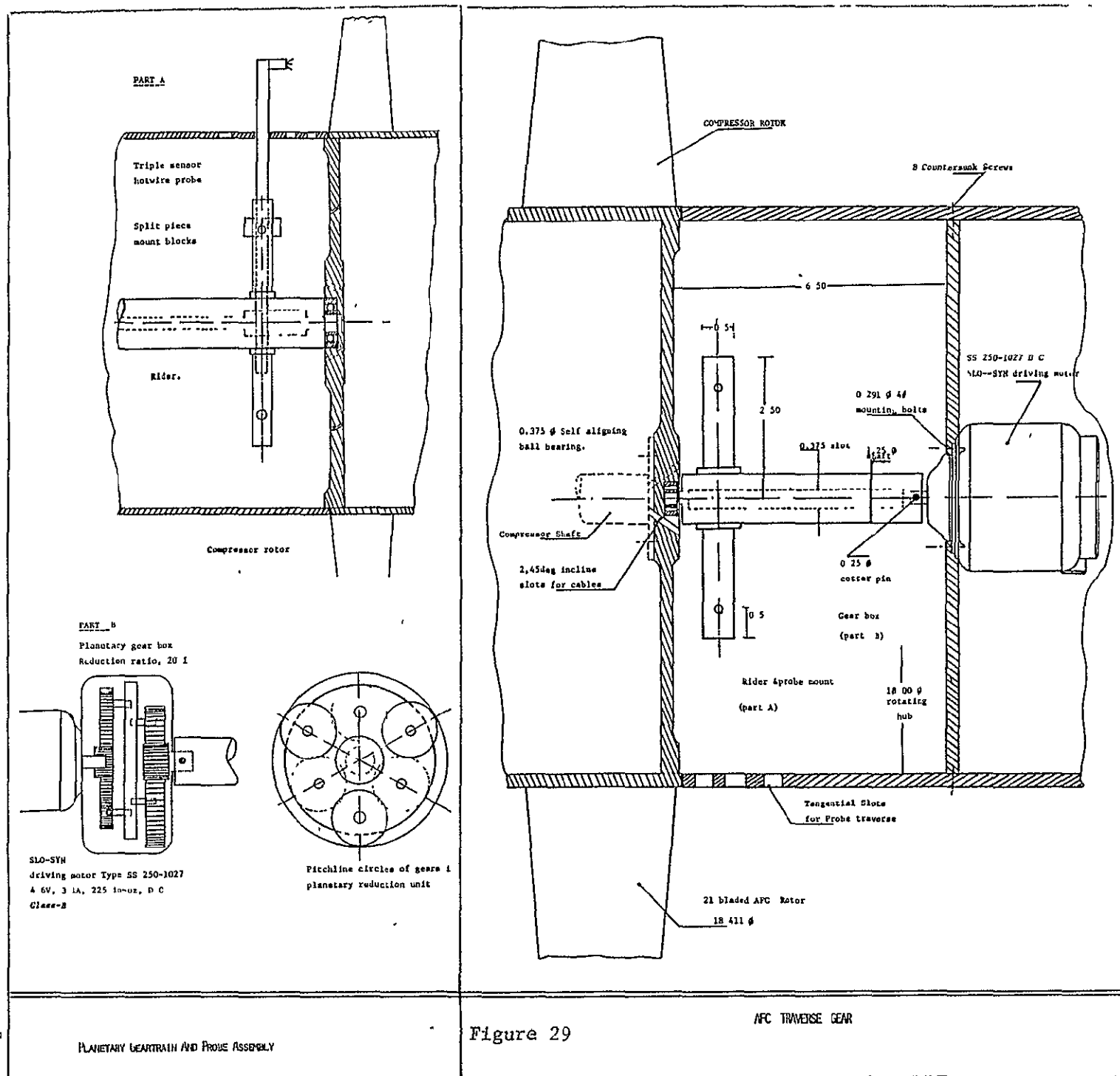


Figure 28: Static pressure distribution across the wake ( $x/c = 0.021$ ,  $R/R_T = 0.860$ ,  $i = 10^\circ$ )





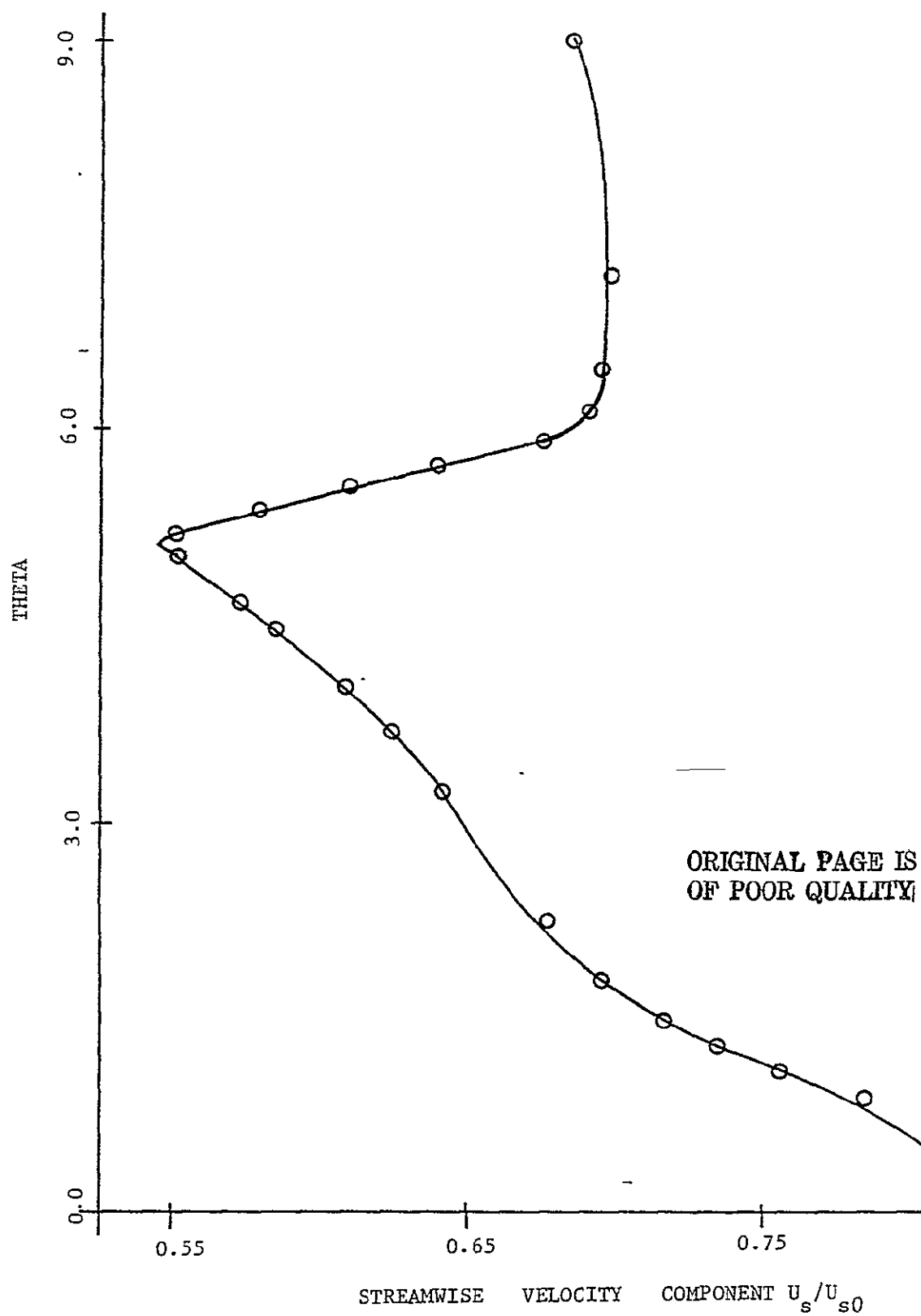


Figure 30:

AFC ROTOR WAKE,  $X/C=0.290$ ,  $R/RT=0.453$ ,  $\phi=0.53$

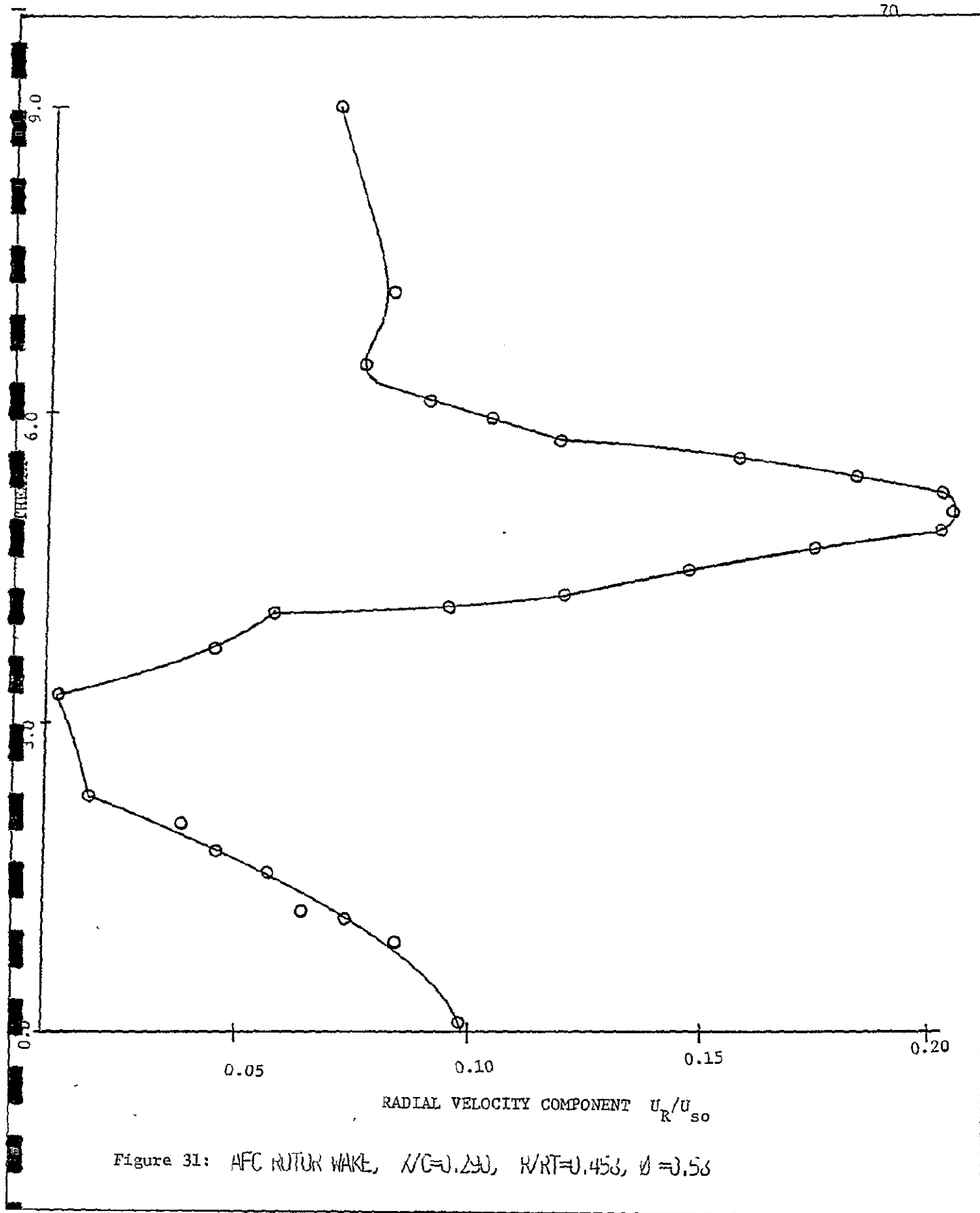


Figure 31: AFC ROTOR WAKE,  $\lambda/C=0.290$ ,  $W/RT=0.453$ ,  $\phi=0.53$

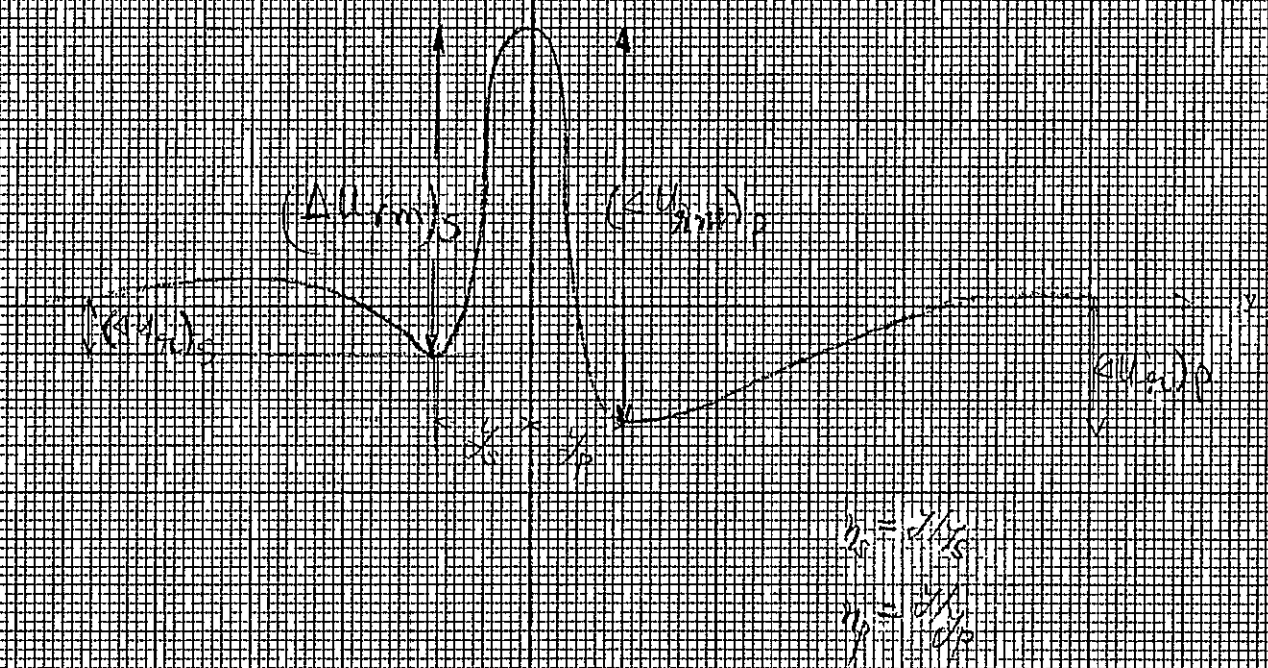
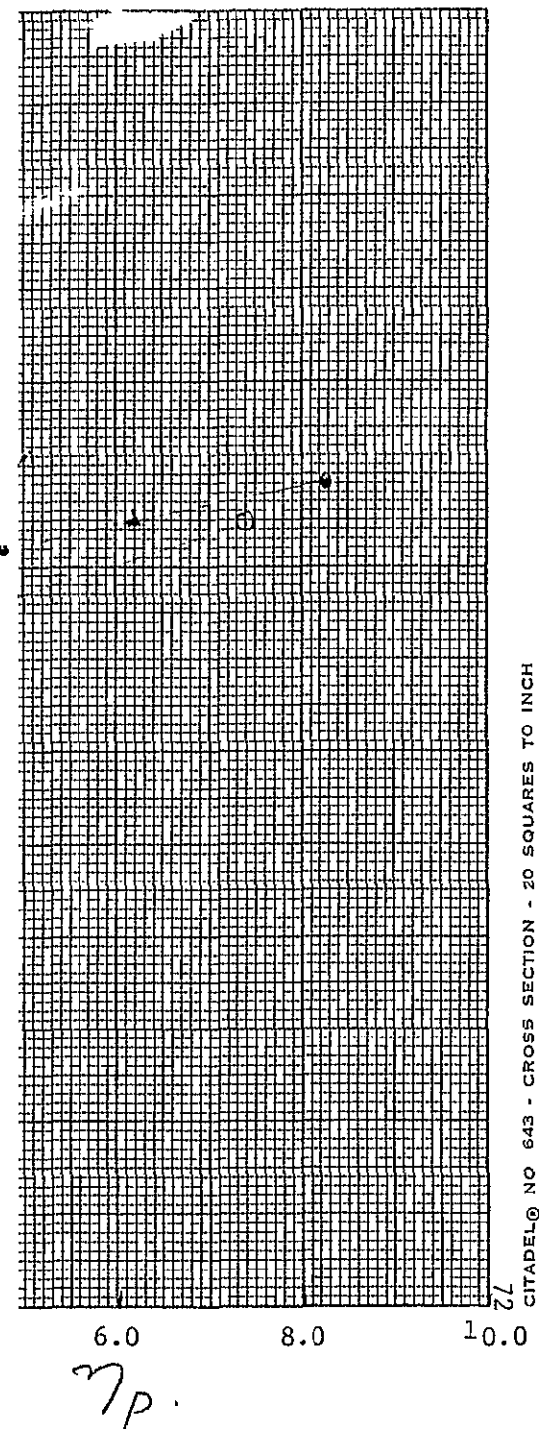
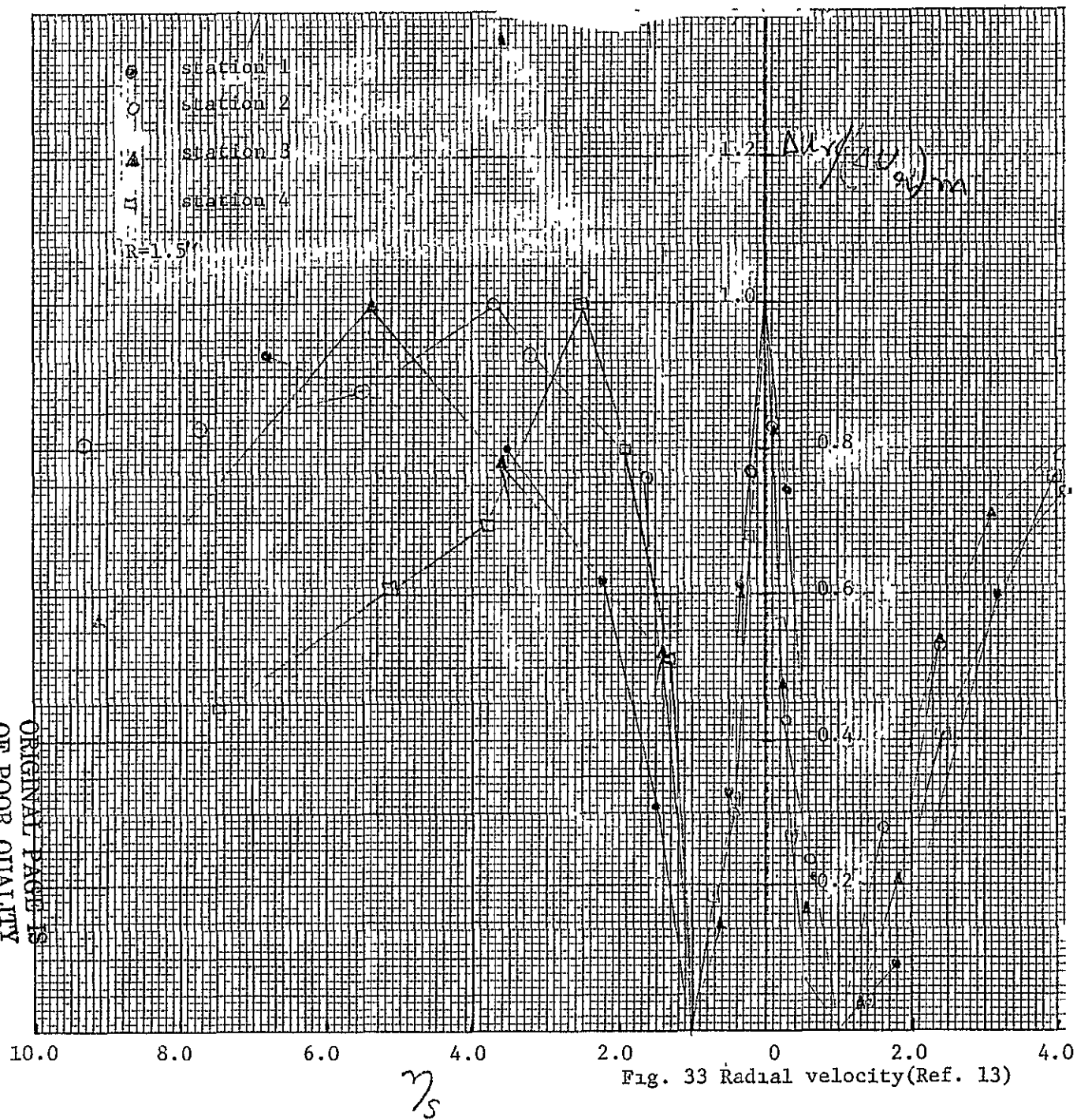


Fig. 32 Typical radial velocity profile

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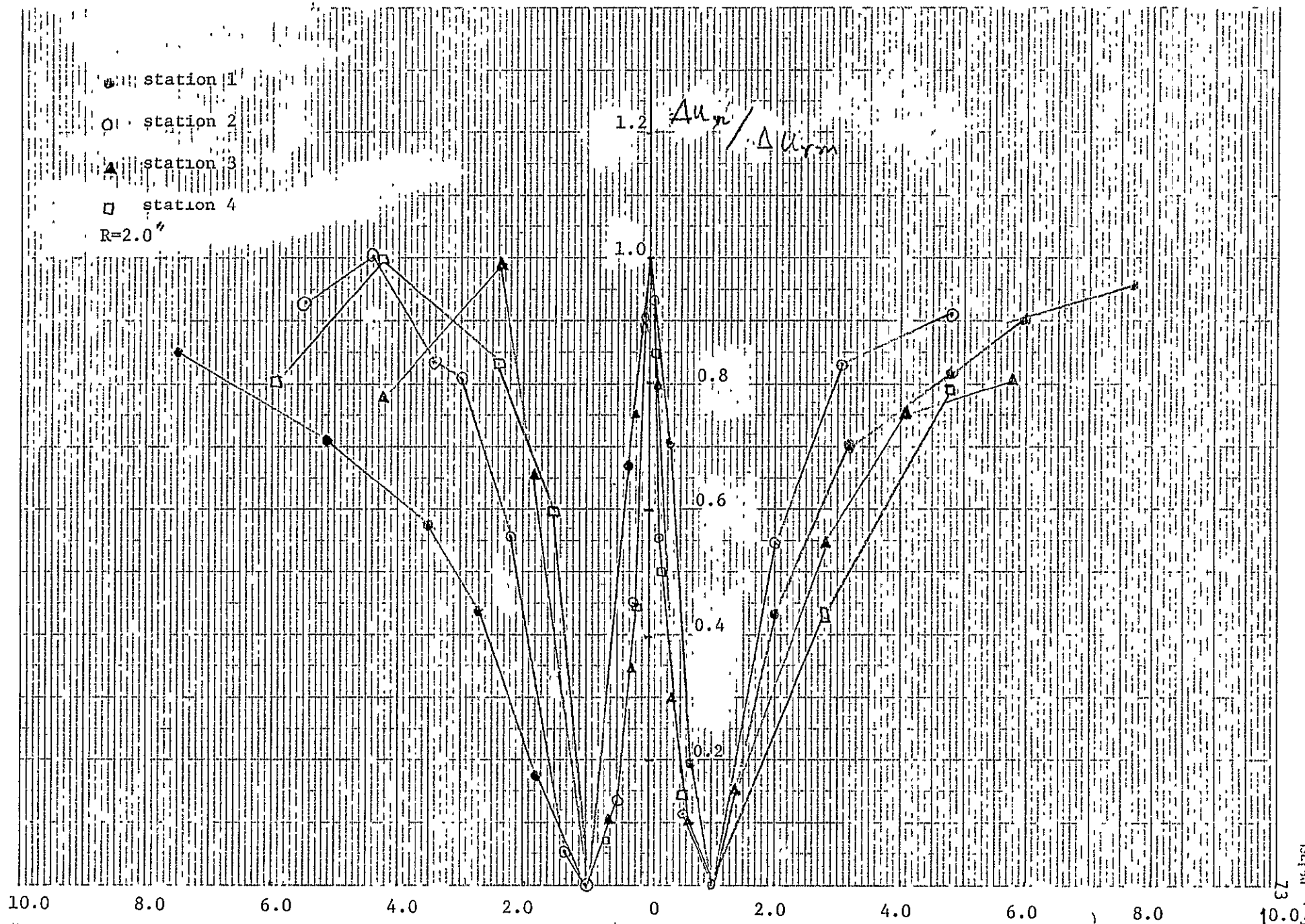


Fig. 34 Radial velocity(Ref. 13)

- 40 % PHH (passage height from hub)
- 50 % PHH
- 60 % PHH
- Gaussian distribution

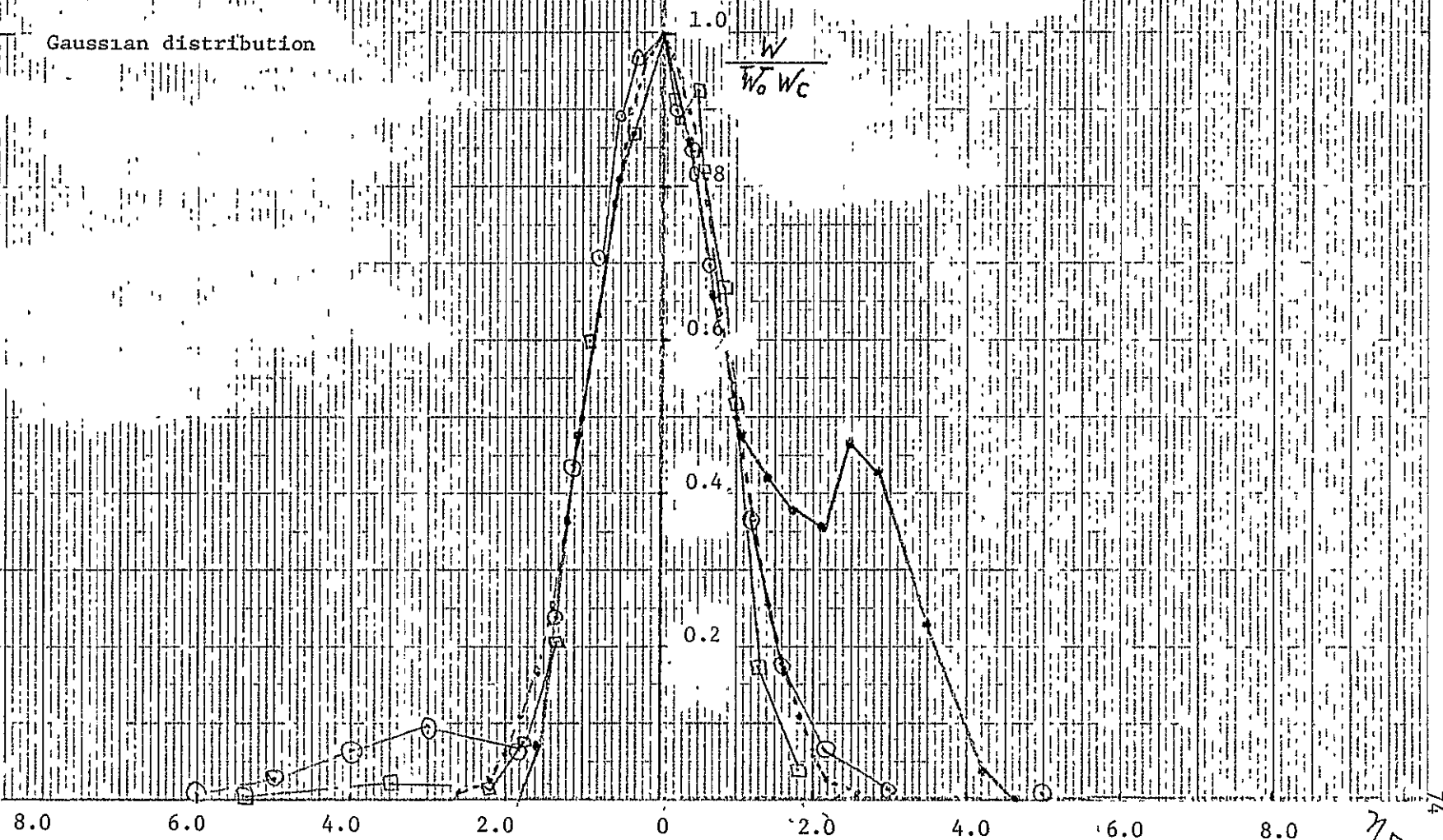


Fig. 35 Axial velocity (Ref. 18)



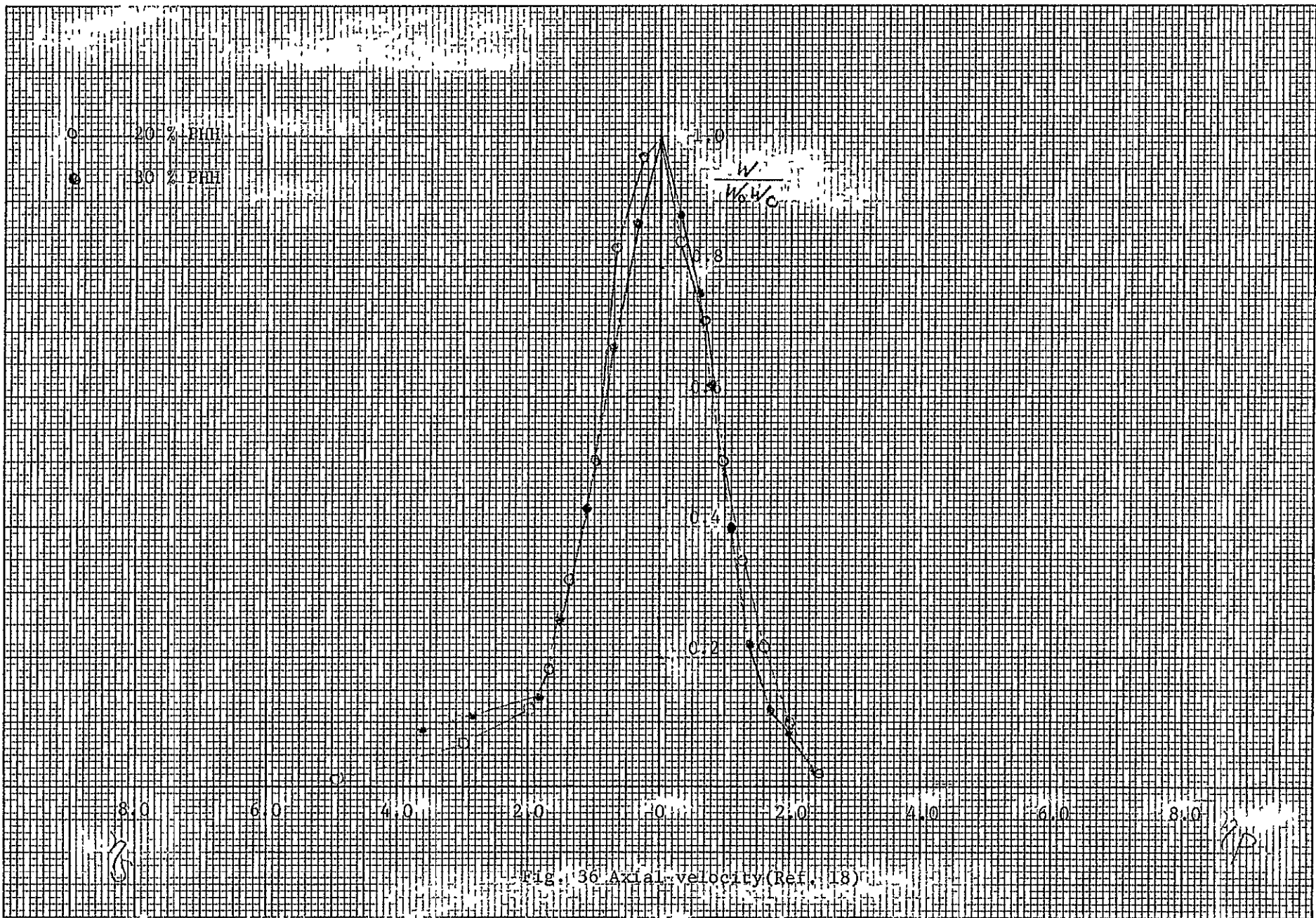


Fig. 36 Axial velocity (Ref. 18)



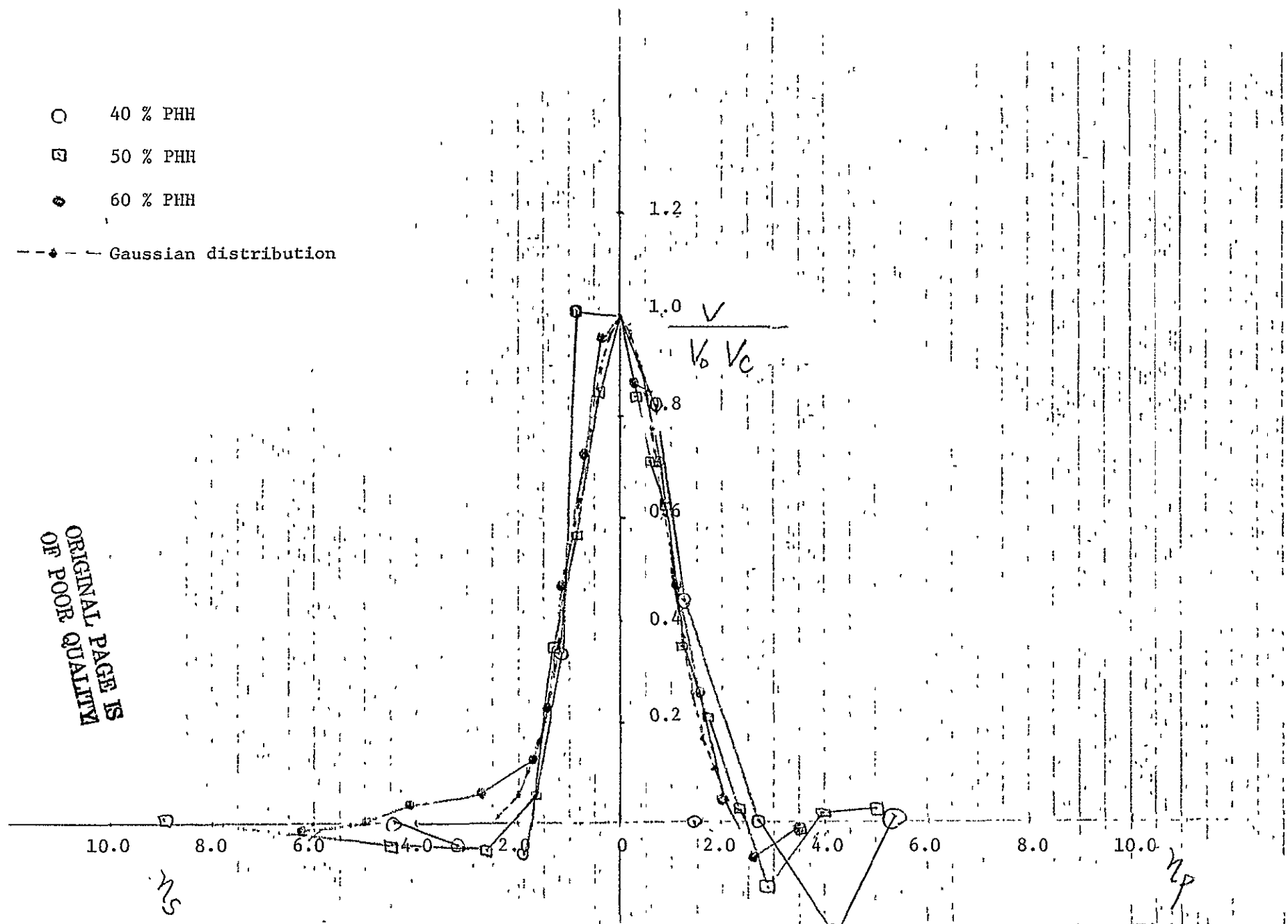
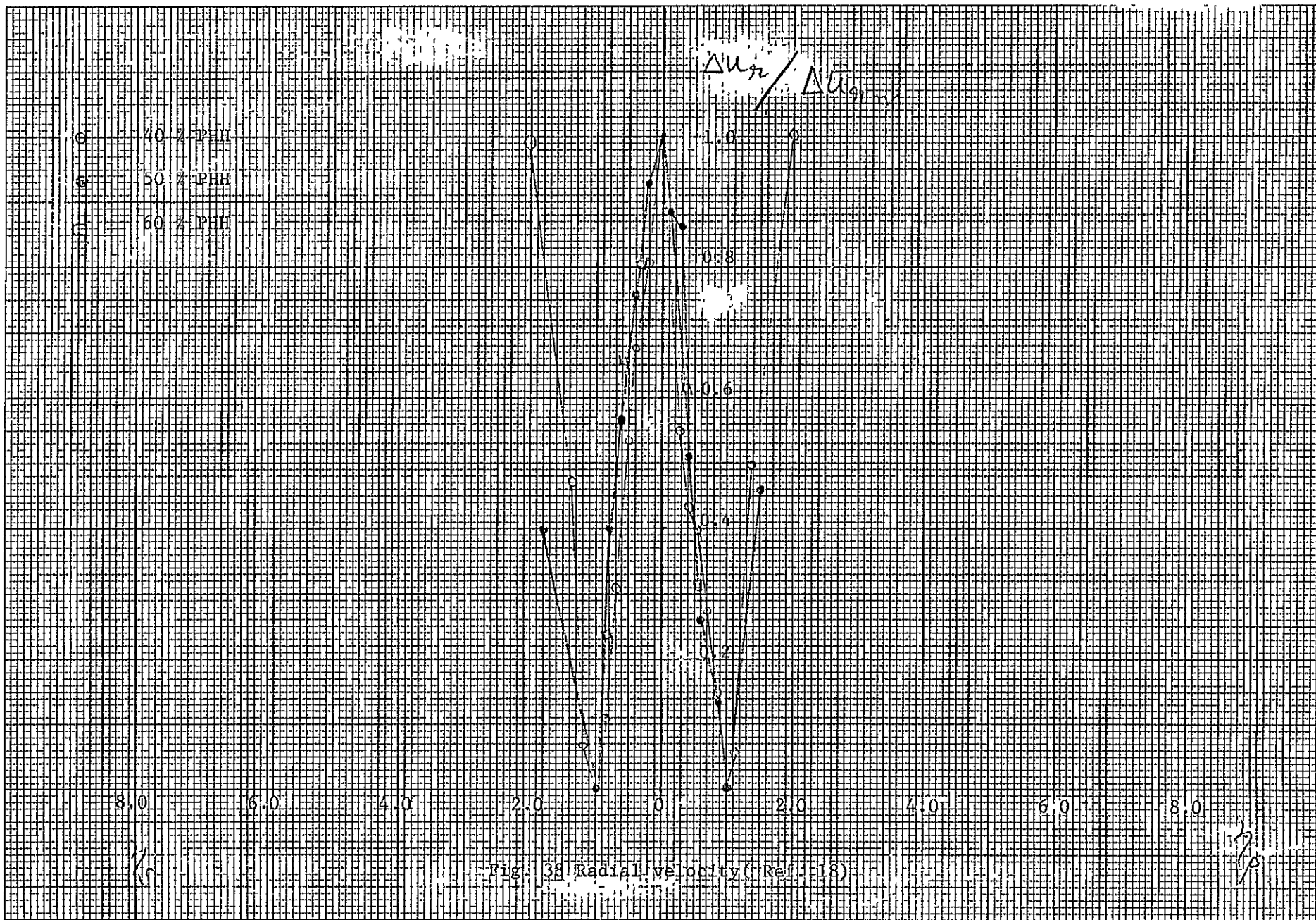


Fig. 37 Absolute tangential velocity(Ref. 18)



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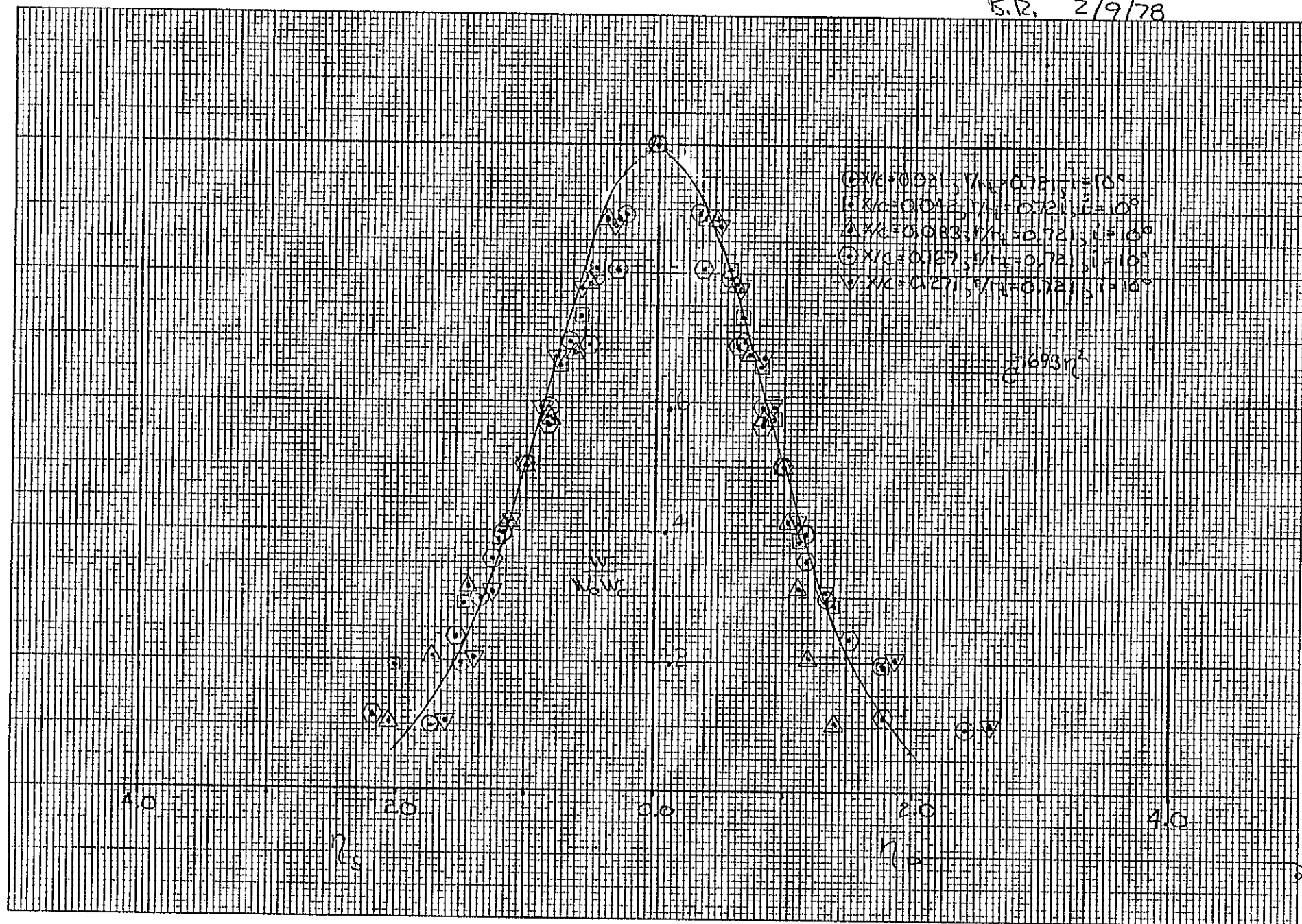


Figure 39: AXIAL VELOCITY SIMILARITY PROFILE -ROTATING PROBE DATA

B.R. 2/9/78

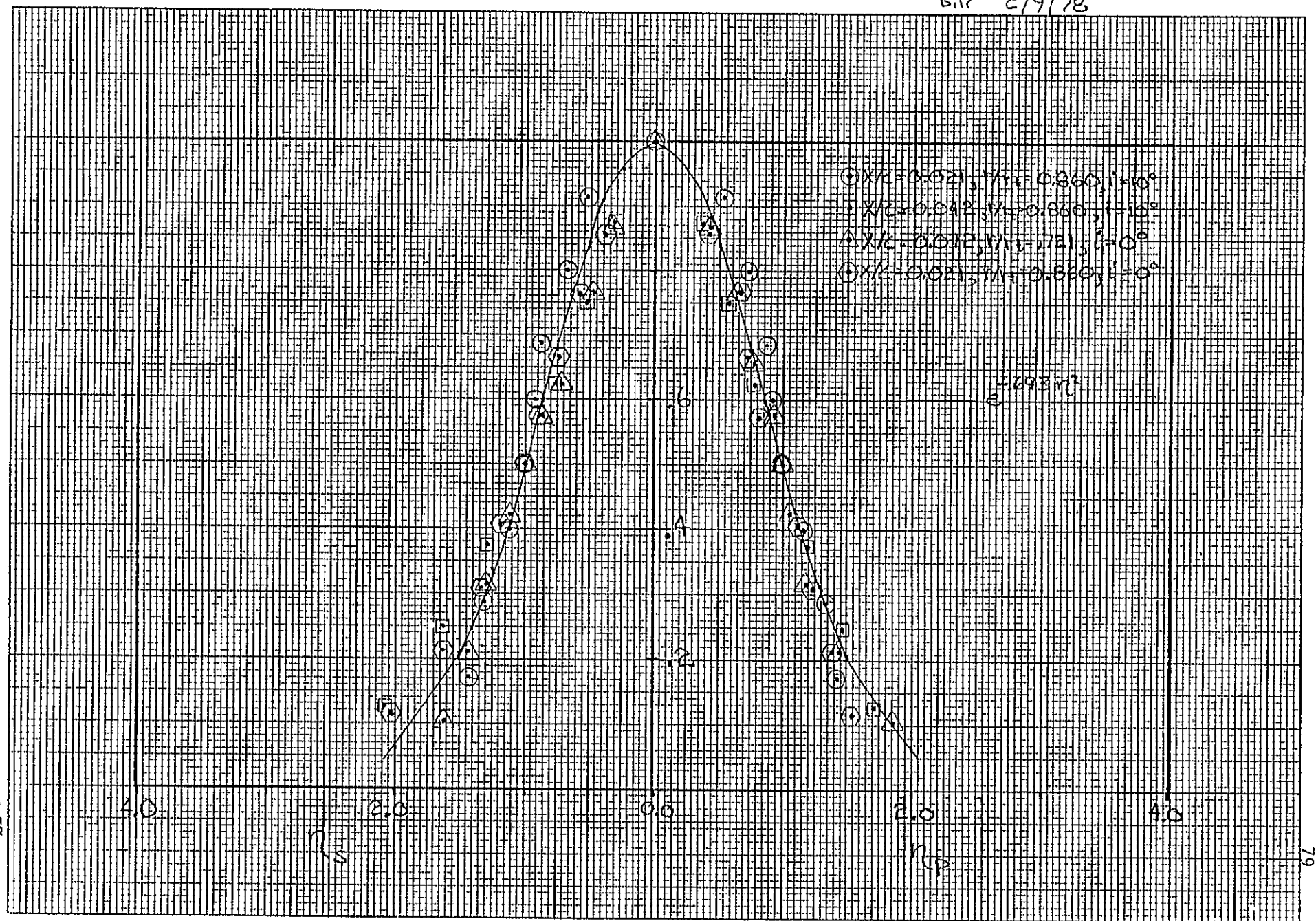


Figure 40: AXIAL VELOCITY SIMILARITY PROFILE — ROTATING PROBE DATA

B.R. 2/11/78

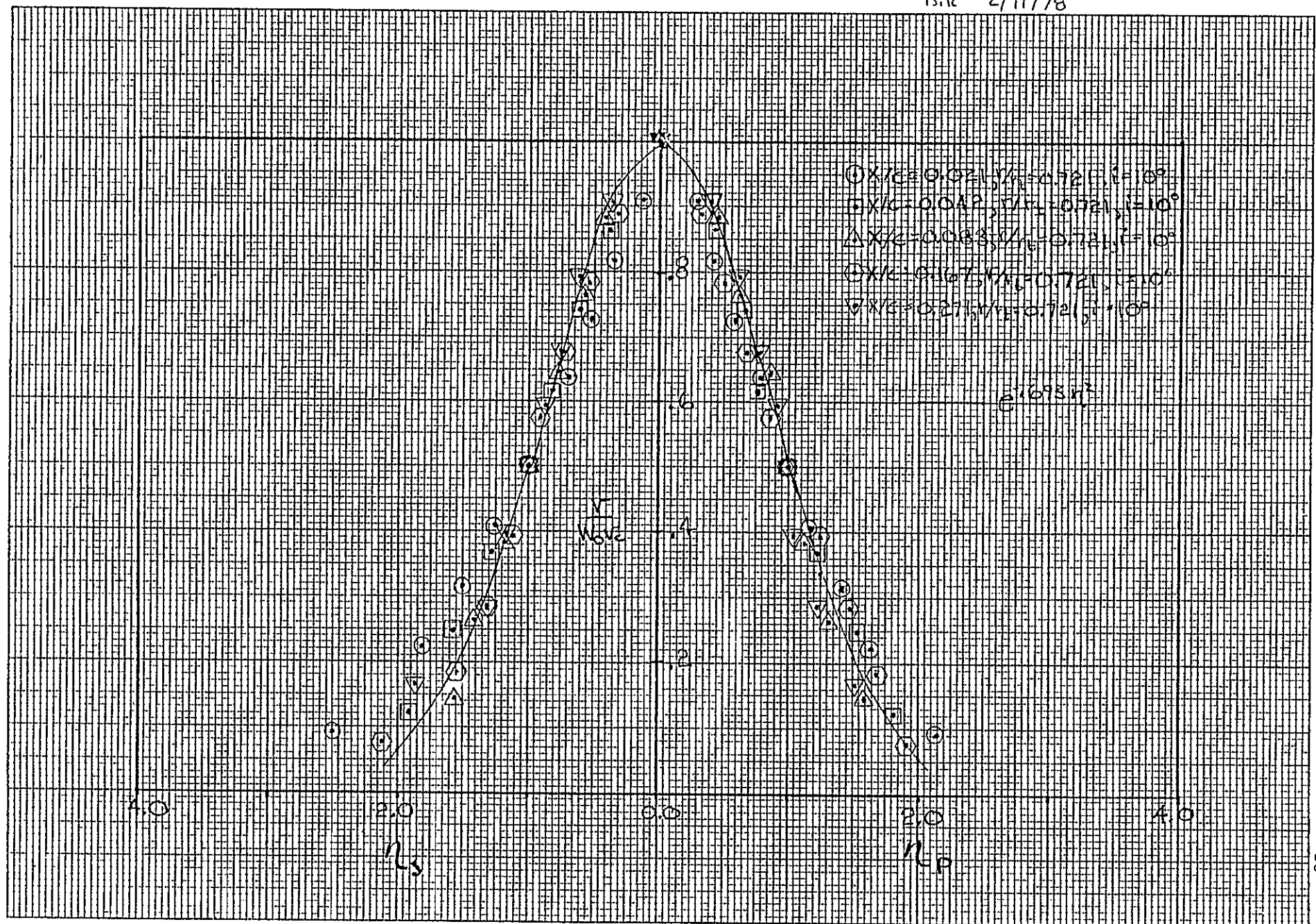


Figure 41: TANGENTIAL VELOCITY SIMILARITY PROFILE - ROTATING PROBE DATA



B.R. 2/11/78

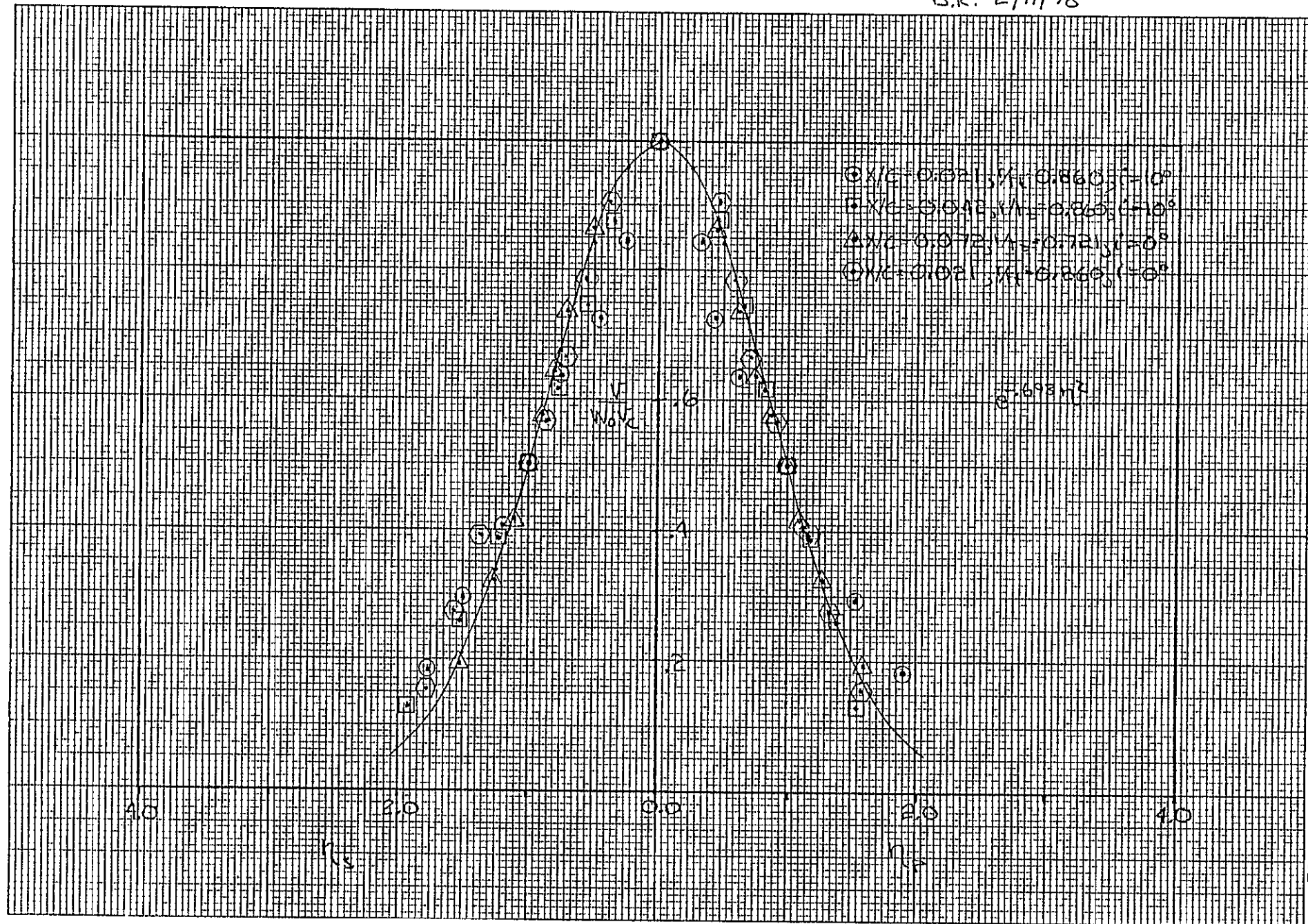


Figure 42: TANGENTIAL VELOCITY SIMILARITY PROFILE - ROTATING PROBE DATA

B.P 2/5/78

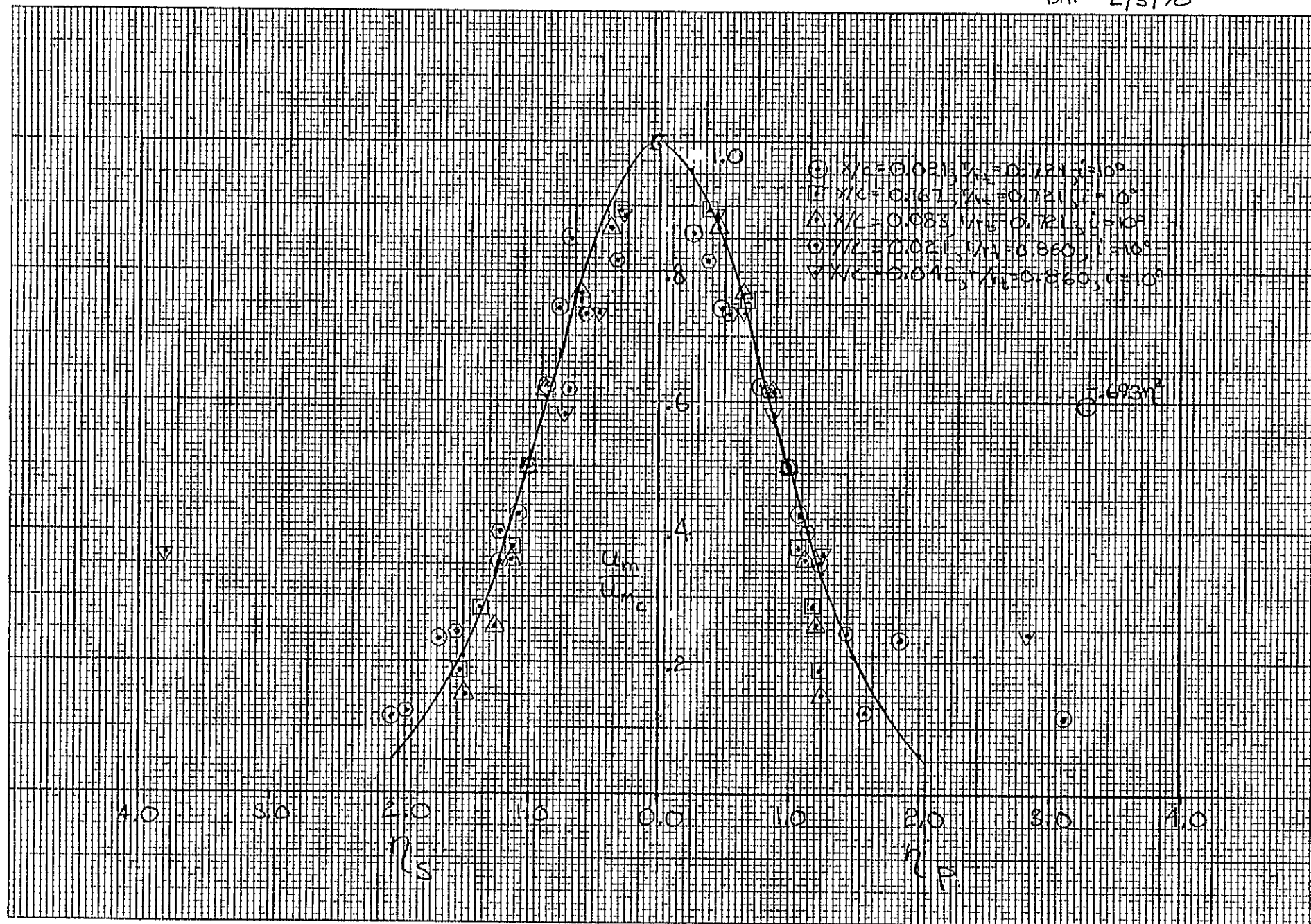
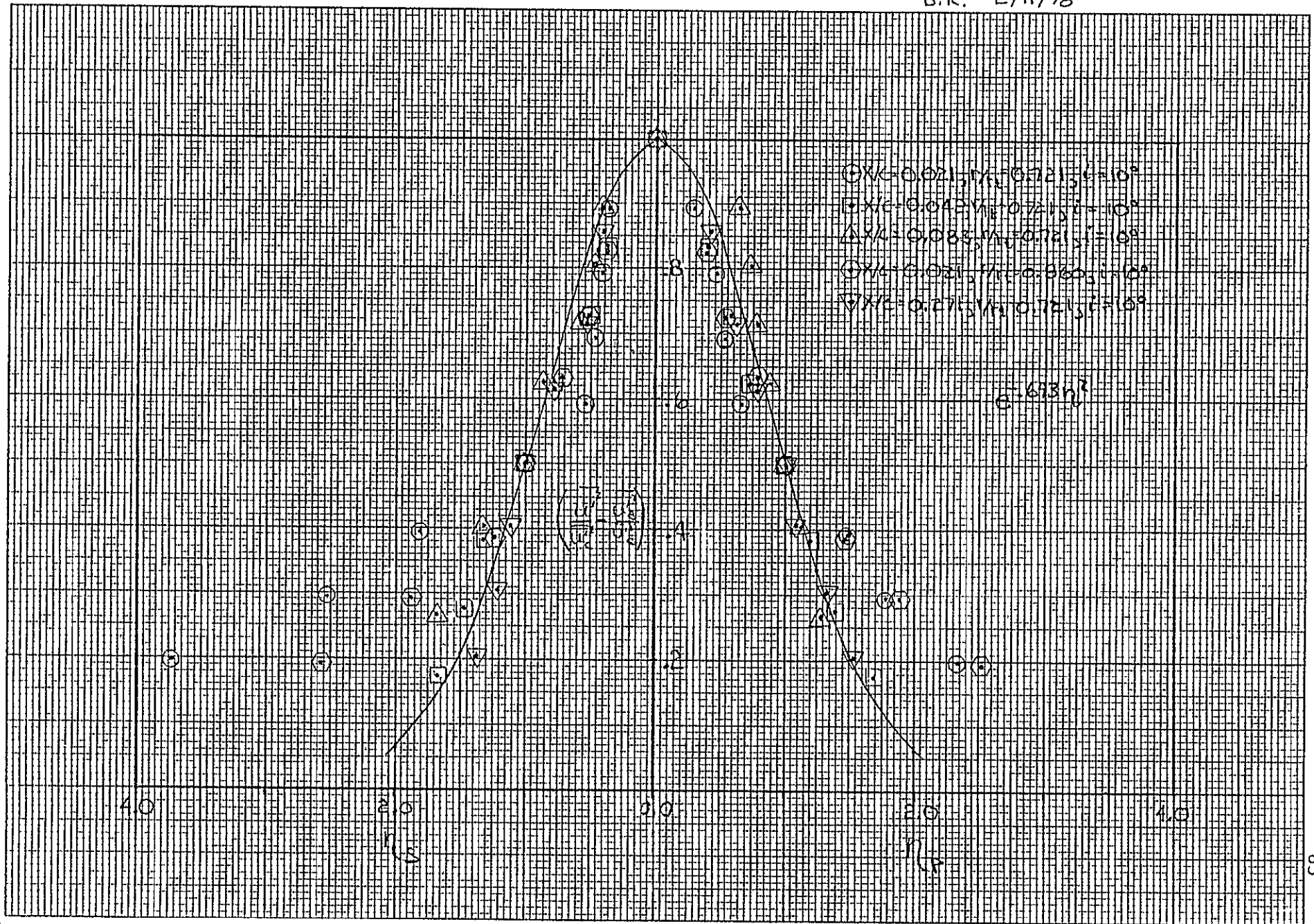


Figure 43: RADIAL VELOCITY SIMILARITY PROFILE

B.R. 2/11/78

CITADEL NO 643 - CROSS SECTION - 20 SQUARES TO INCH



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Figure 44: RADIAL INTENSITY SIMILARITY PROFILES - ROTATING PROBE DATA



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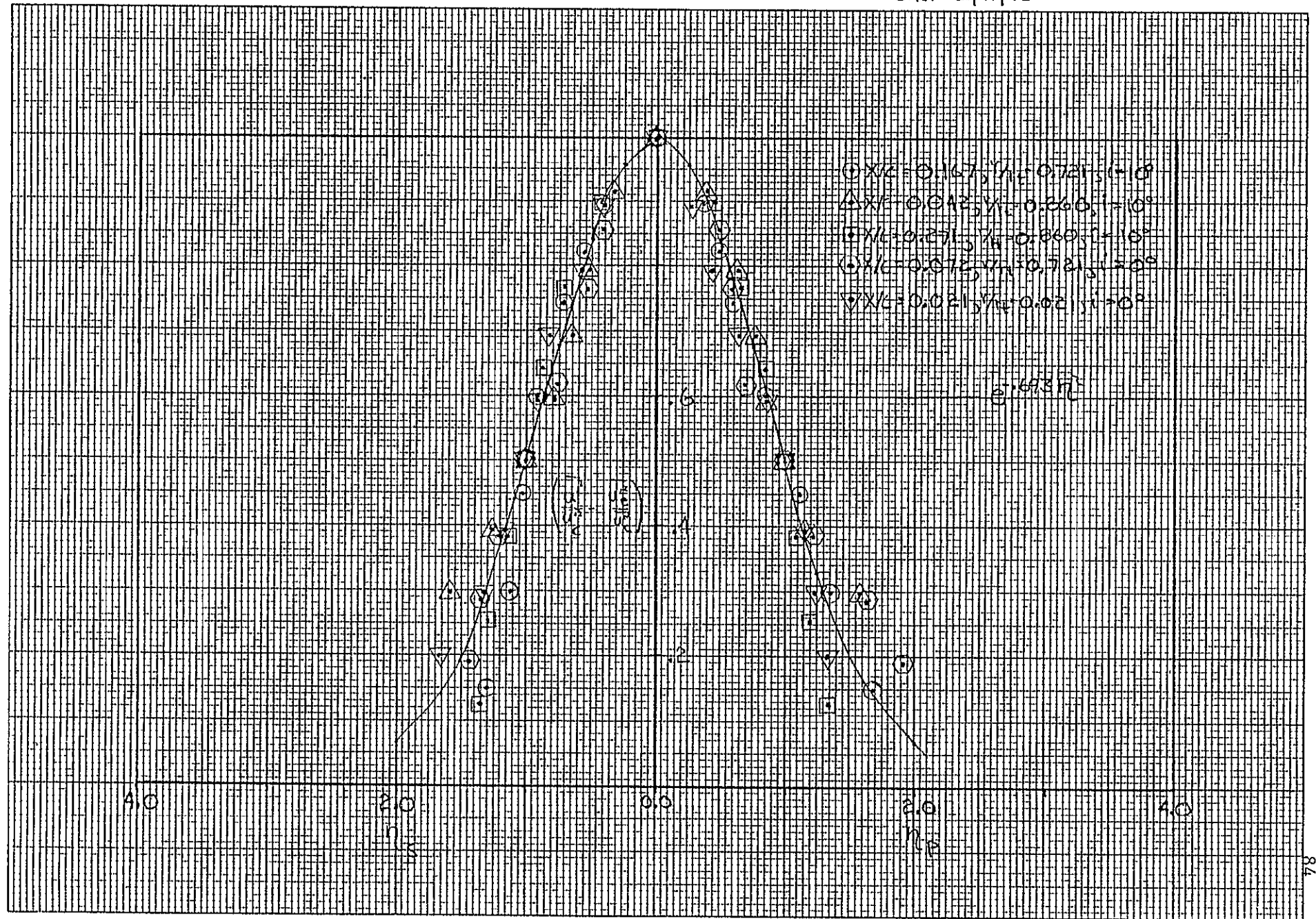


Figure 45: RADIAL INTENSITY SIMILARITY PROFILES - ROTATING PROBE DATA

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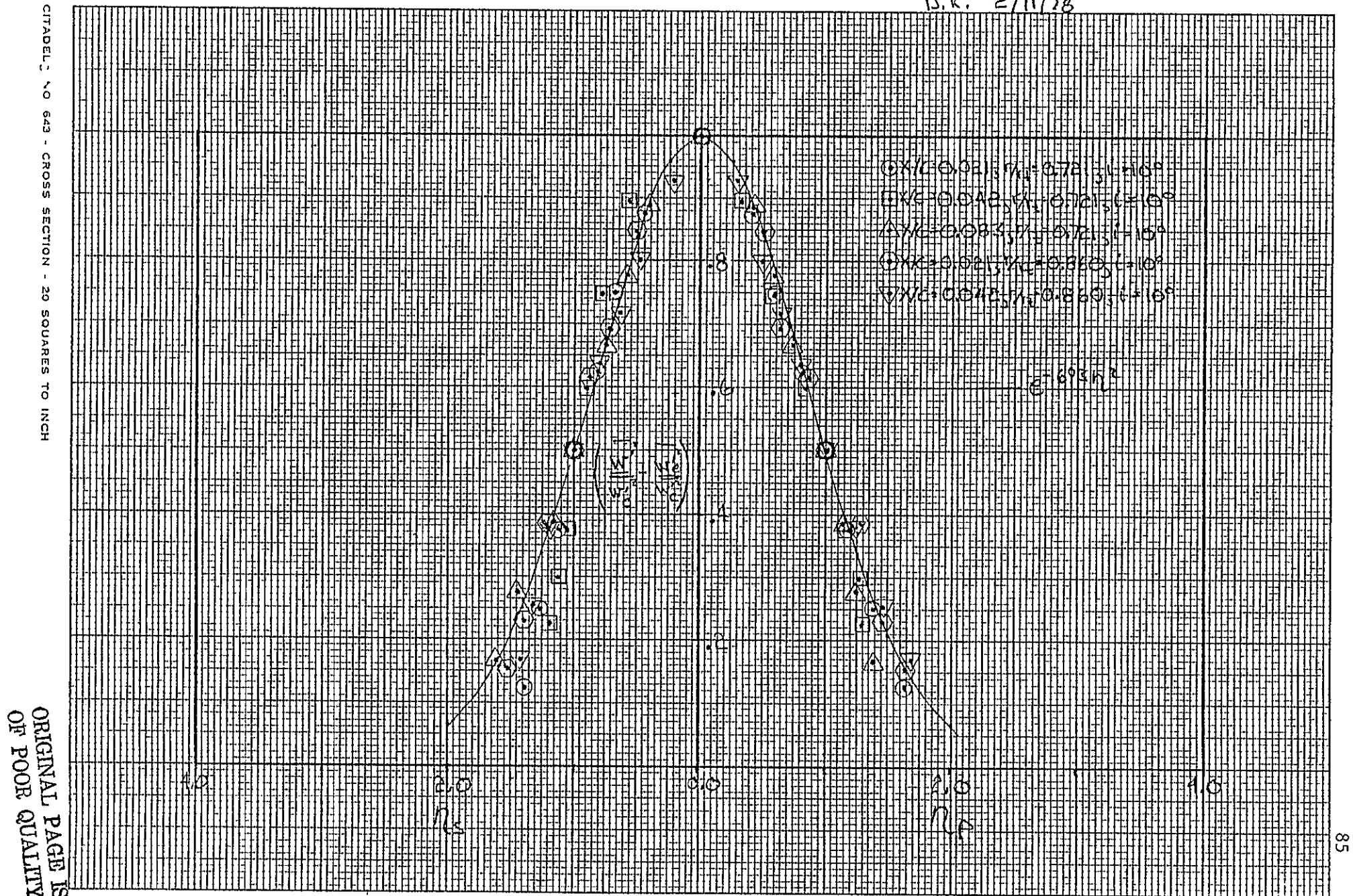


Figure 46: AXIAL INTENSITY SIMILARITY PROFILES - ROTATING PROBE DATA.

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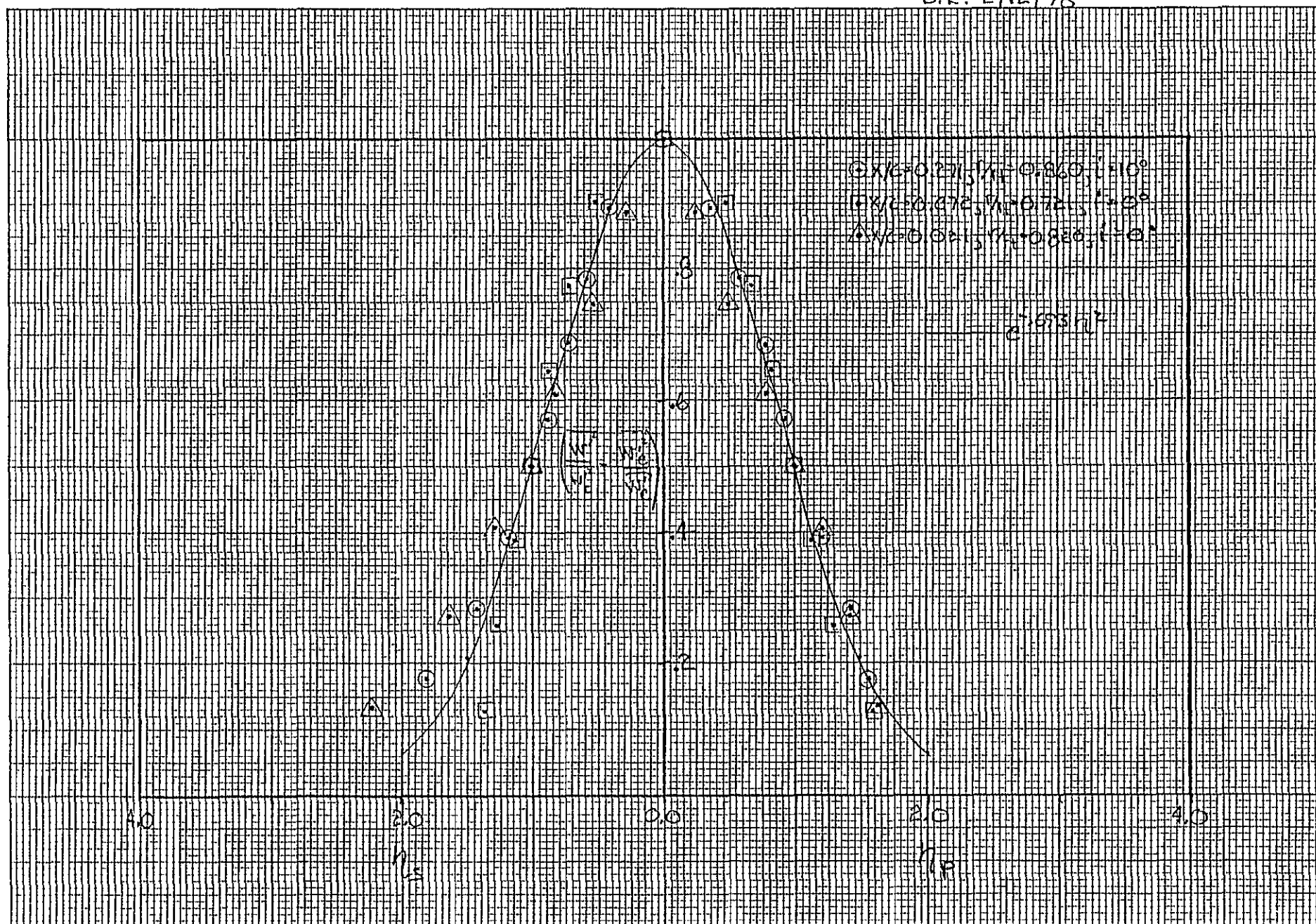


Figure 47: AXIAL INTENSITY SIMILARITY PROFILE - ROTATING PROBE DATA

B.P. 2/12/78

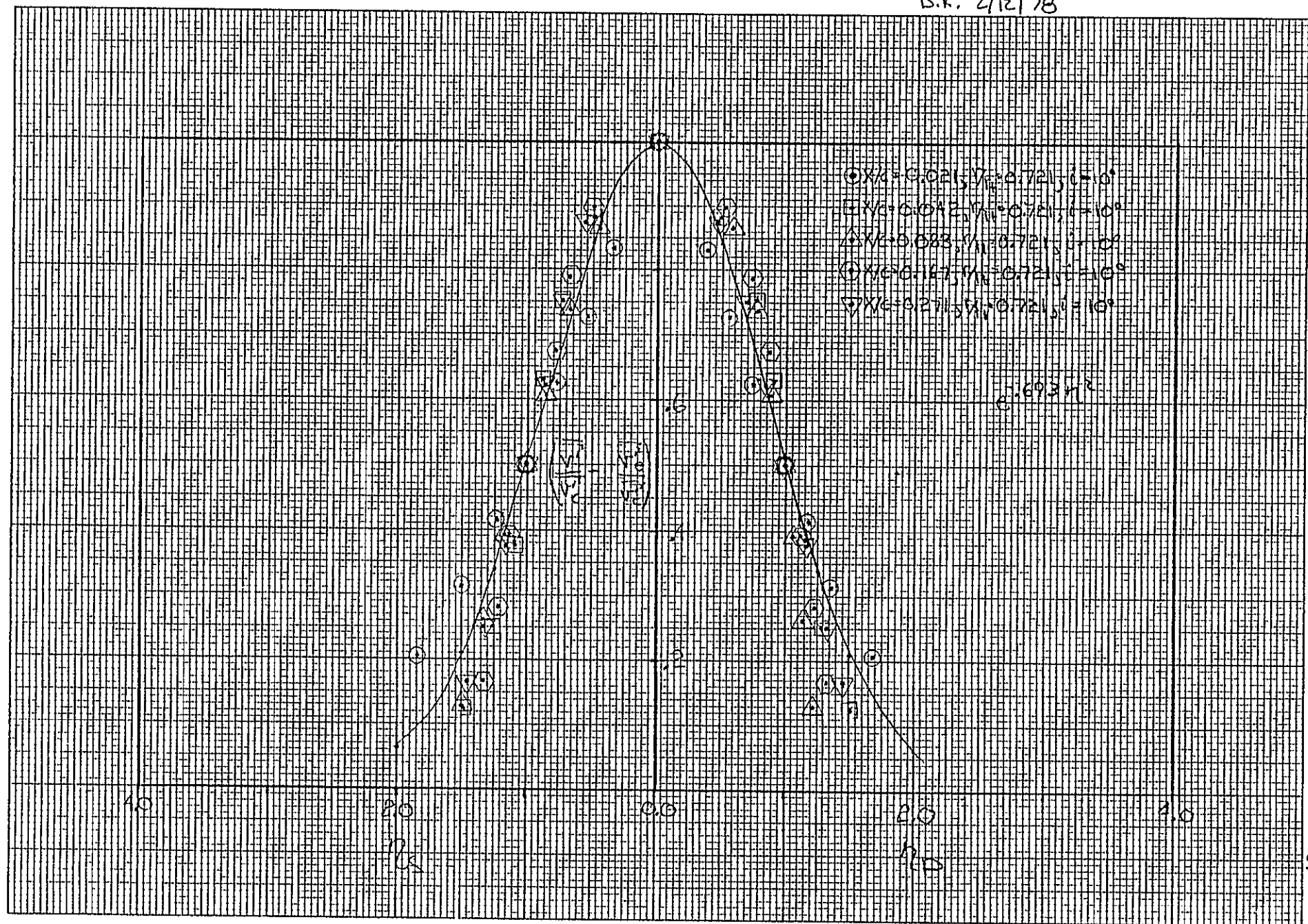


Figure 48: TANGENTIAL INTENSITY SIMILARITY PROFILES - ROTATING PROBE DATA



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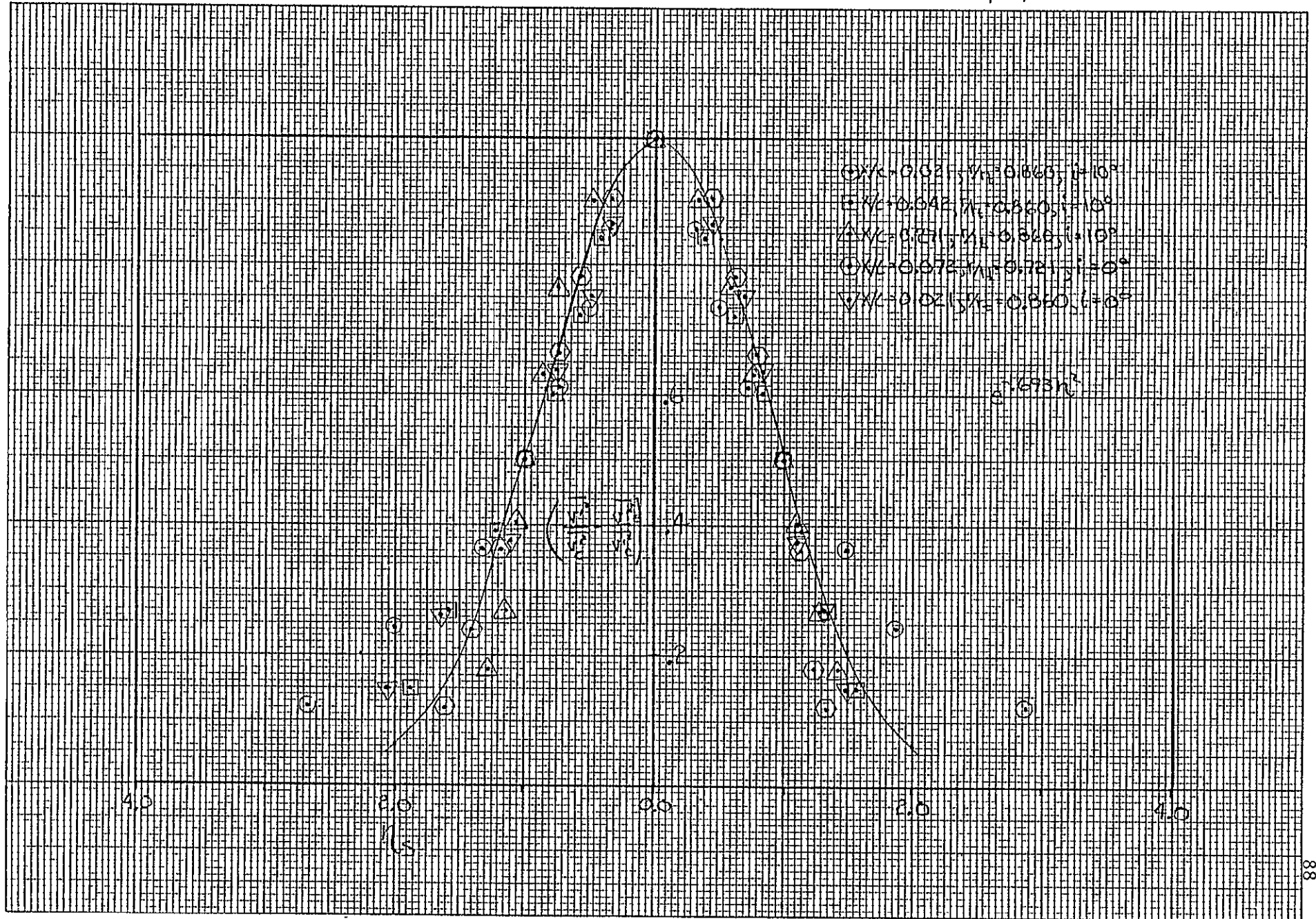


Figure 49: TANGENTIAL INTENSITY SIMILARITY PROFILES - ROTATING PROBE DATA

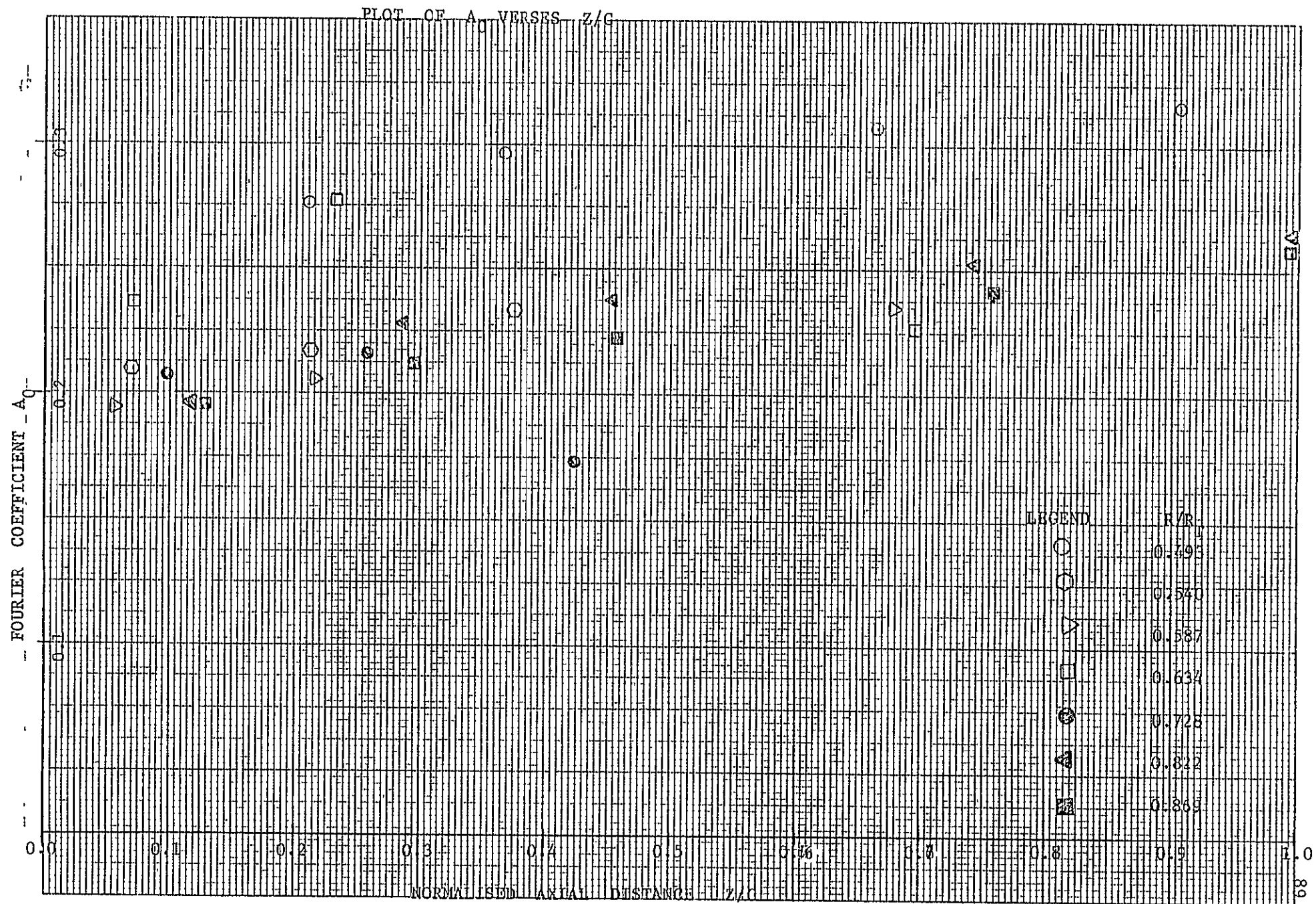


FIG.50

FOURIER COEFFICIENT  $A_0$  VERSES  $Z/C$

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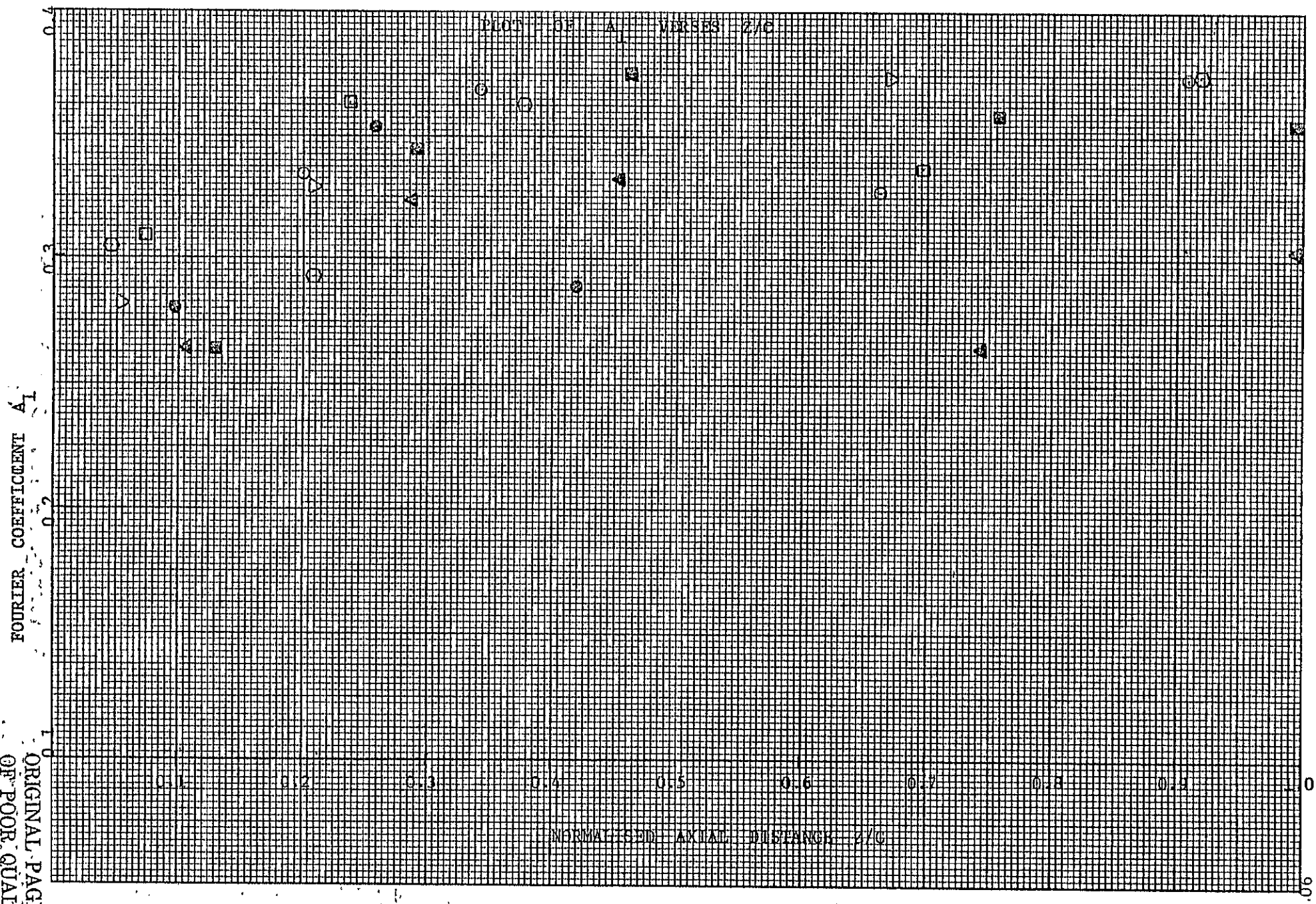


FIG. 51 FOURIER COEFFICIENT  $A_1$  VERSUS  $Z/C$

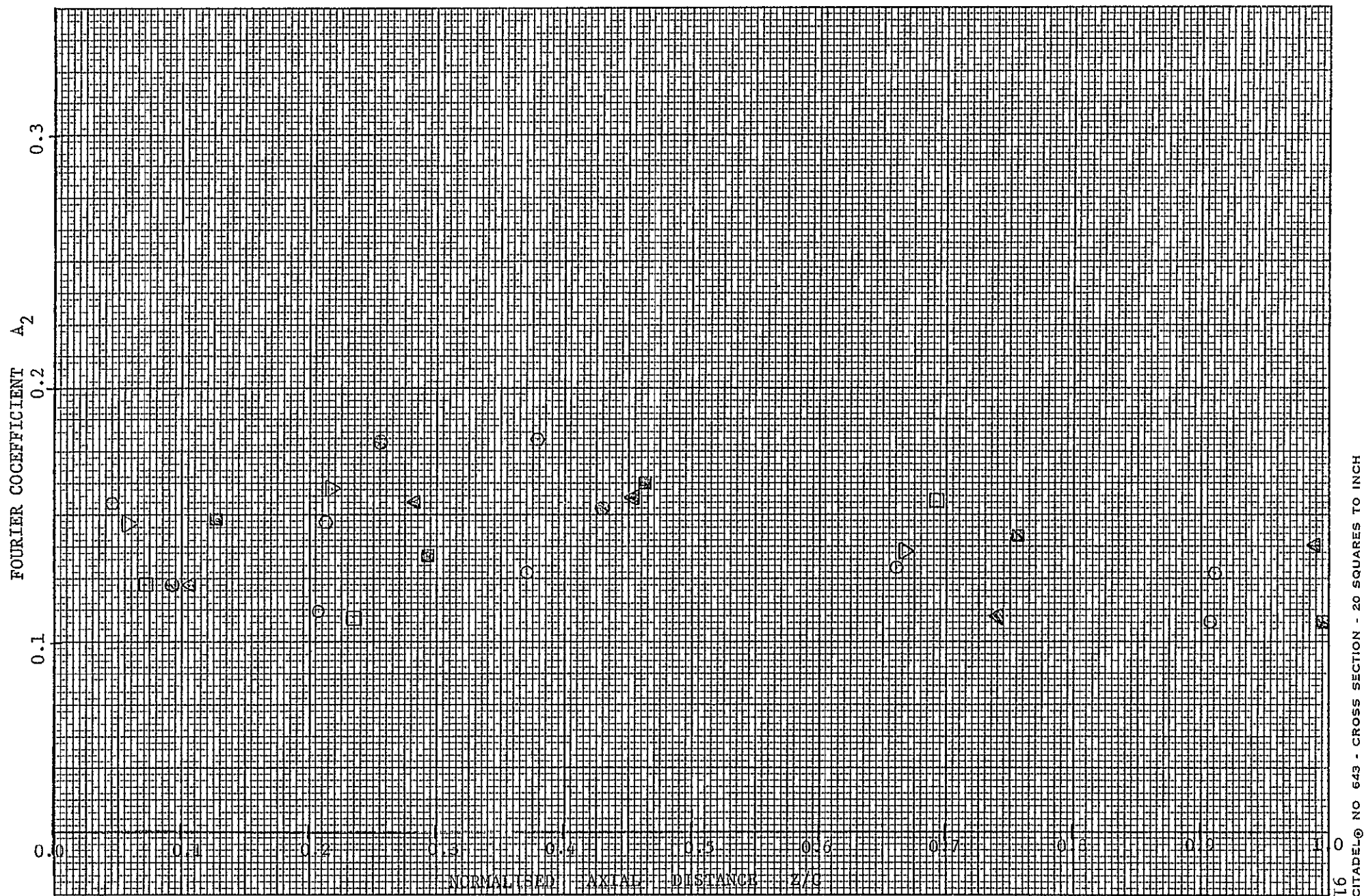


FIG. 52 FOURIER COEFFICIENT  $A_2$  VERSUS  $z/c$



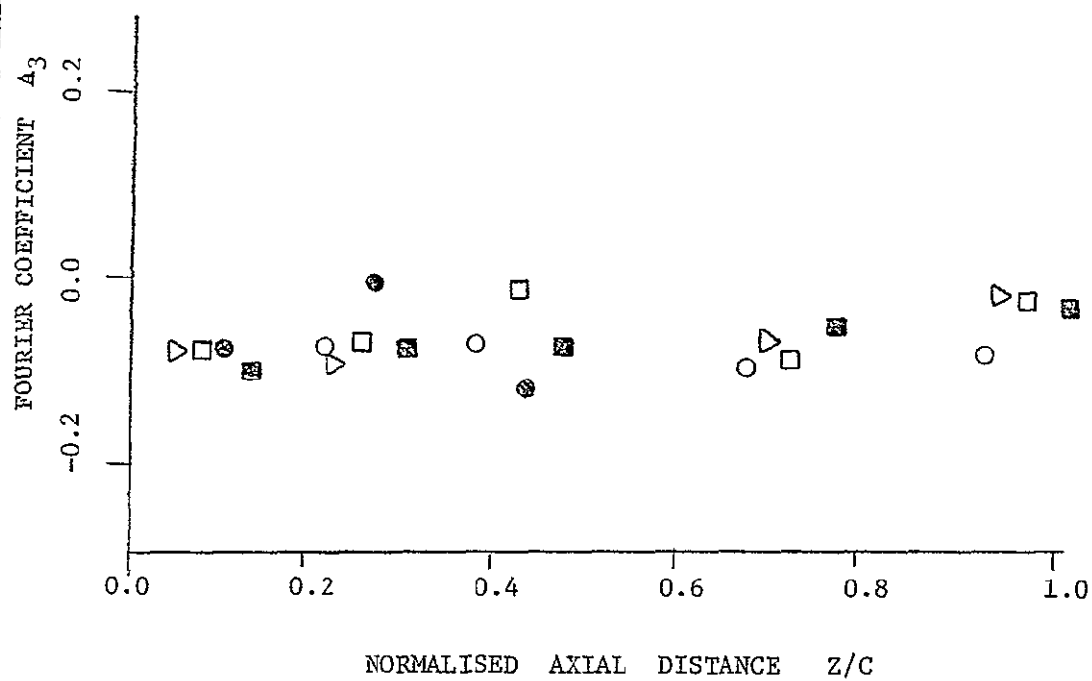


FIG. 53 PLOT OF FOURIER COEFFICIENT  $A_3$  VERSES  $z/c$

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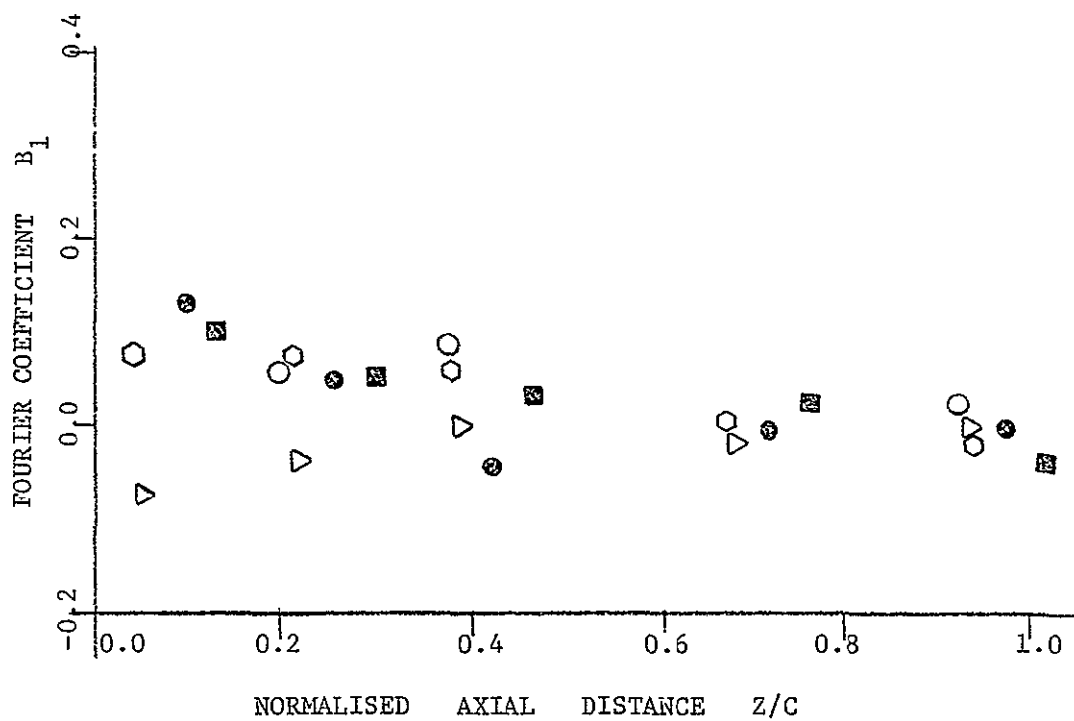


FIG. 54 FOURIER COEFFICIENT  $B_1$  VERSES  $z/c$

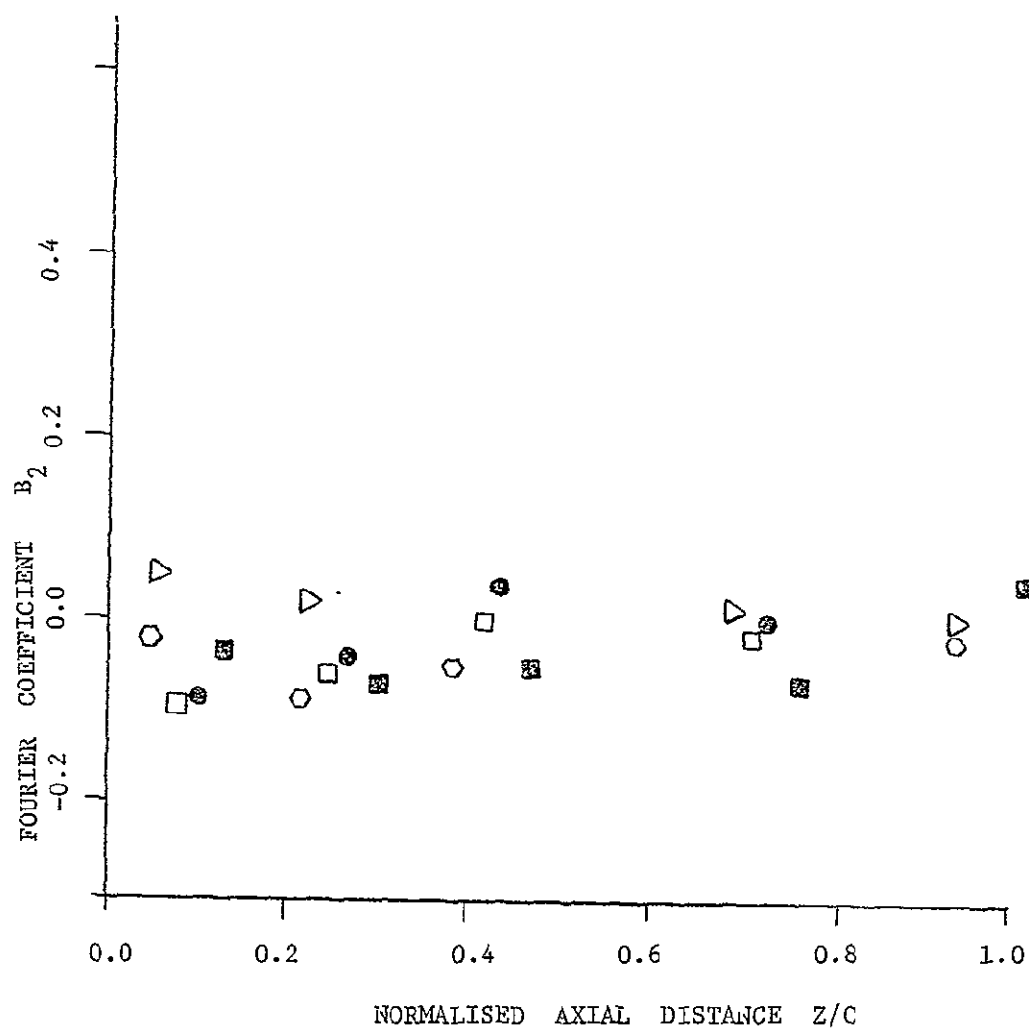


FIG.55 PLOT OF FOURIER COEFFICIENT  $B_2$  VERSES  $Z/C$

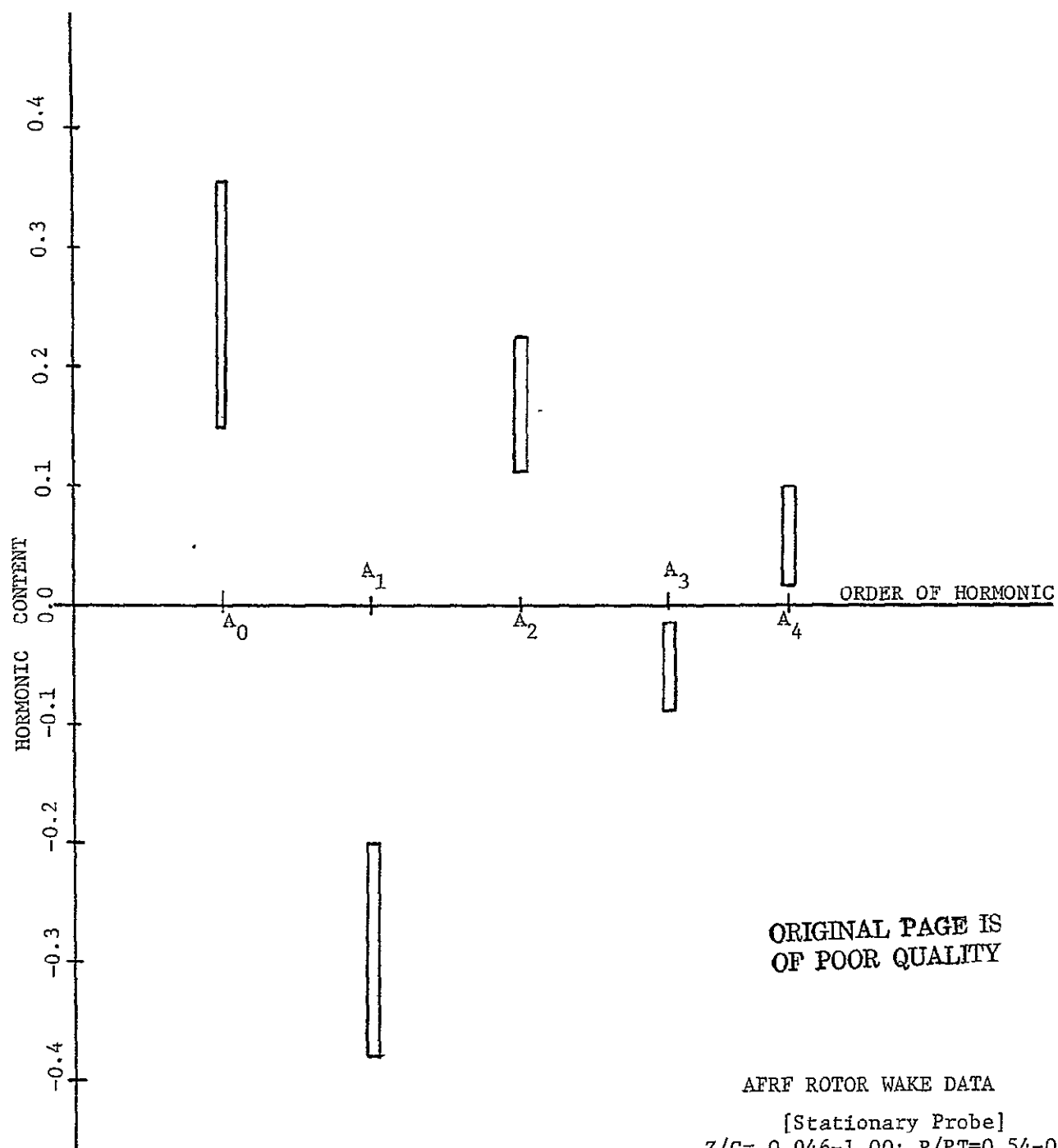


FIG. 56 SCATTER OF HARMONIC CONTENT

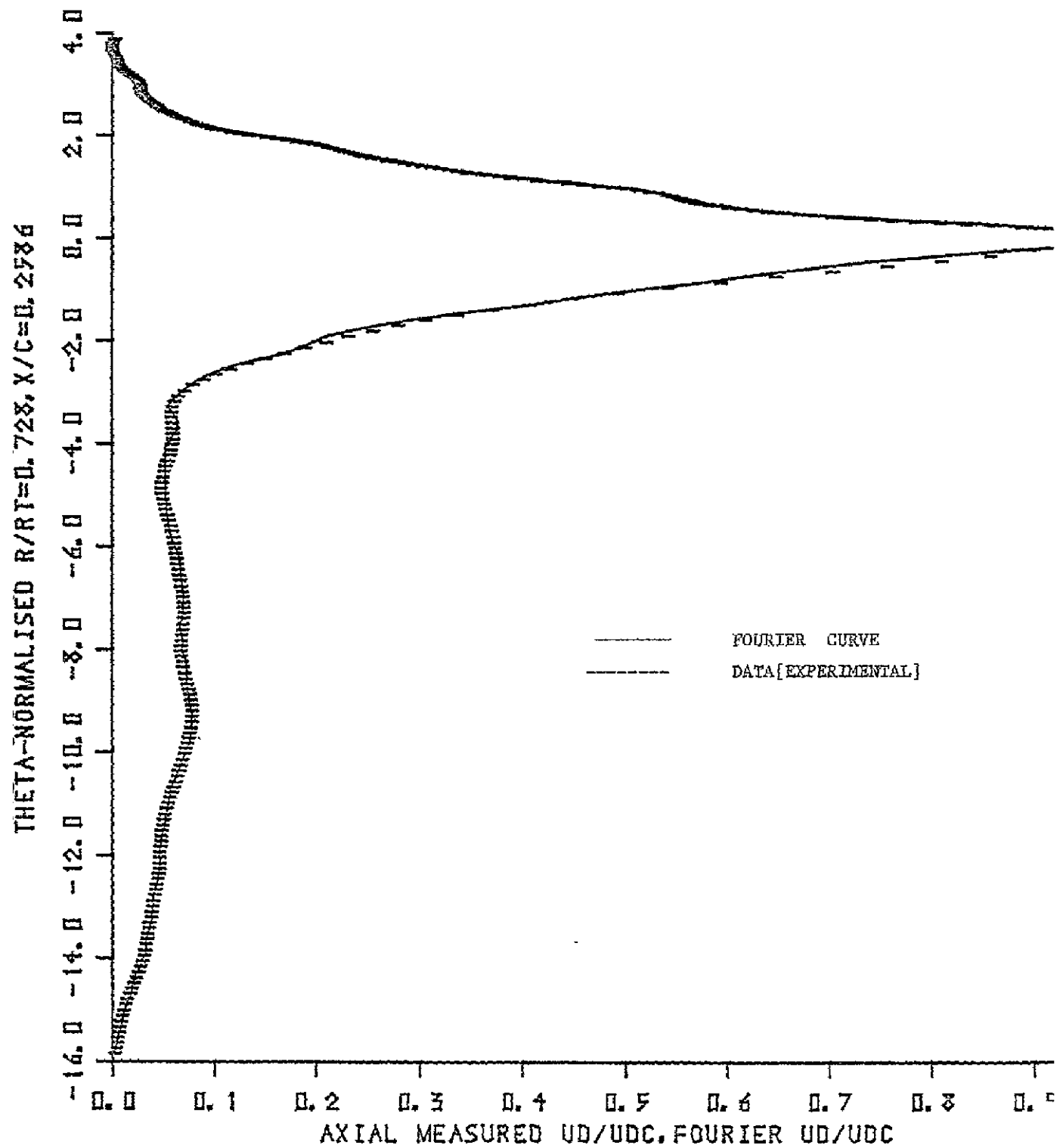


Figure 57: Comparison of Fourier Curve and Data

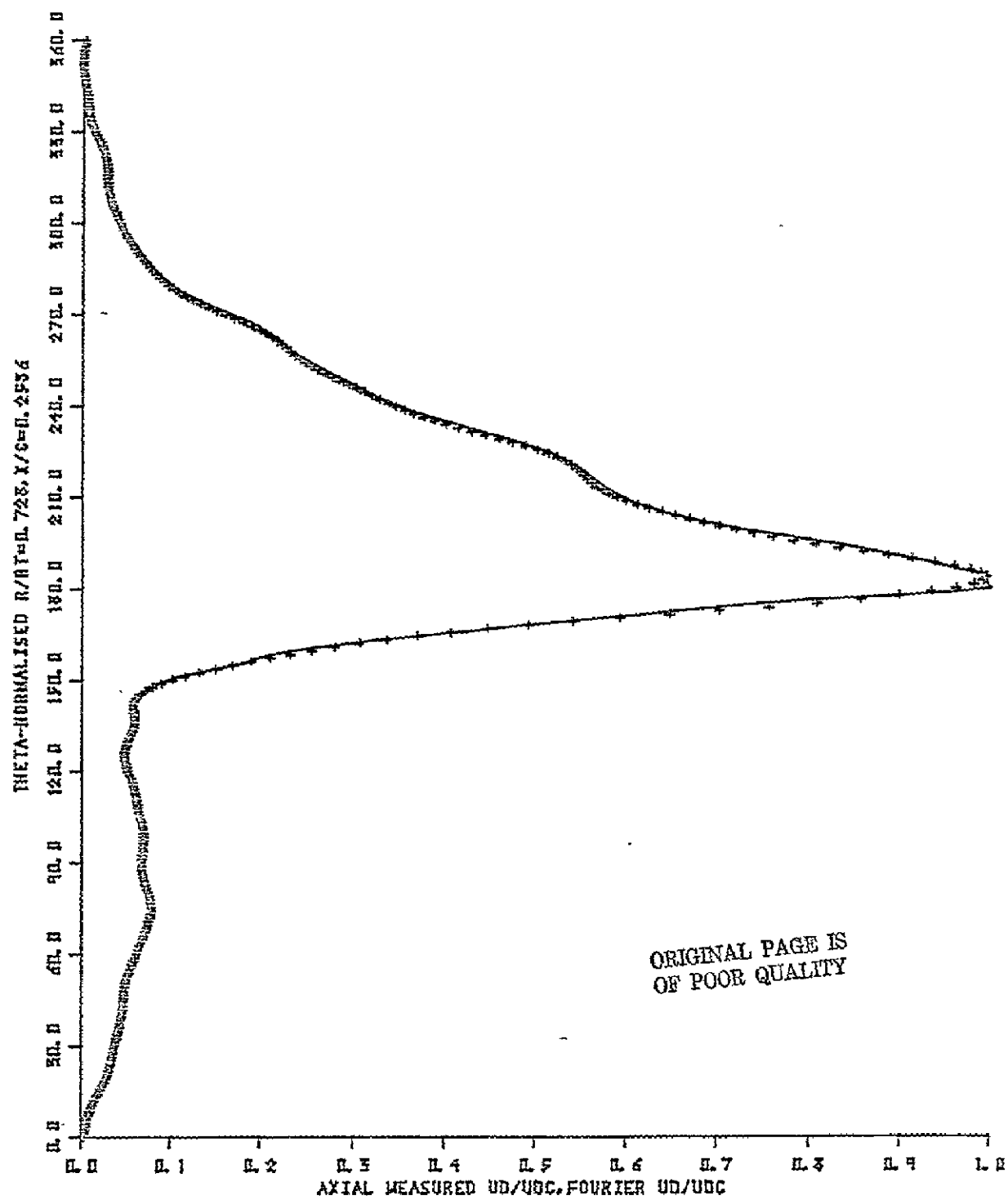


FIG.58 COMPARISON OF FOURIER CURVE AND DATA

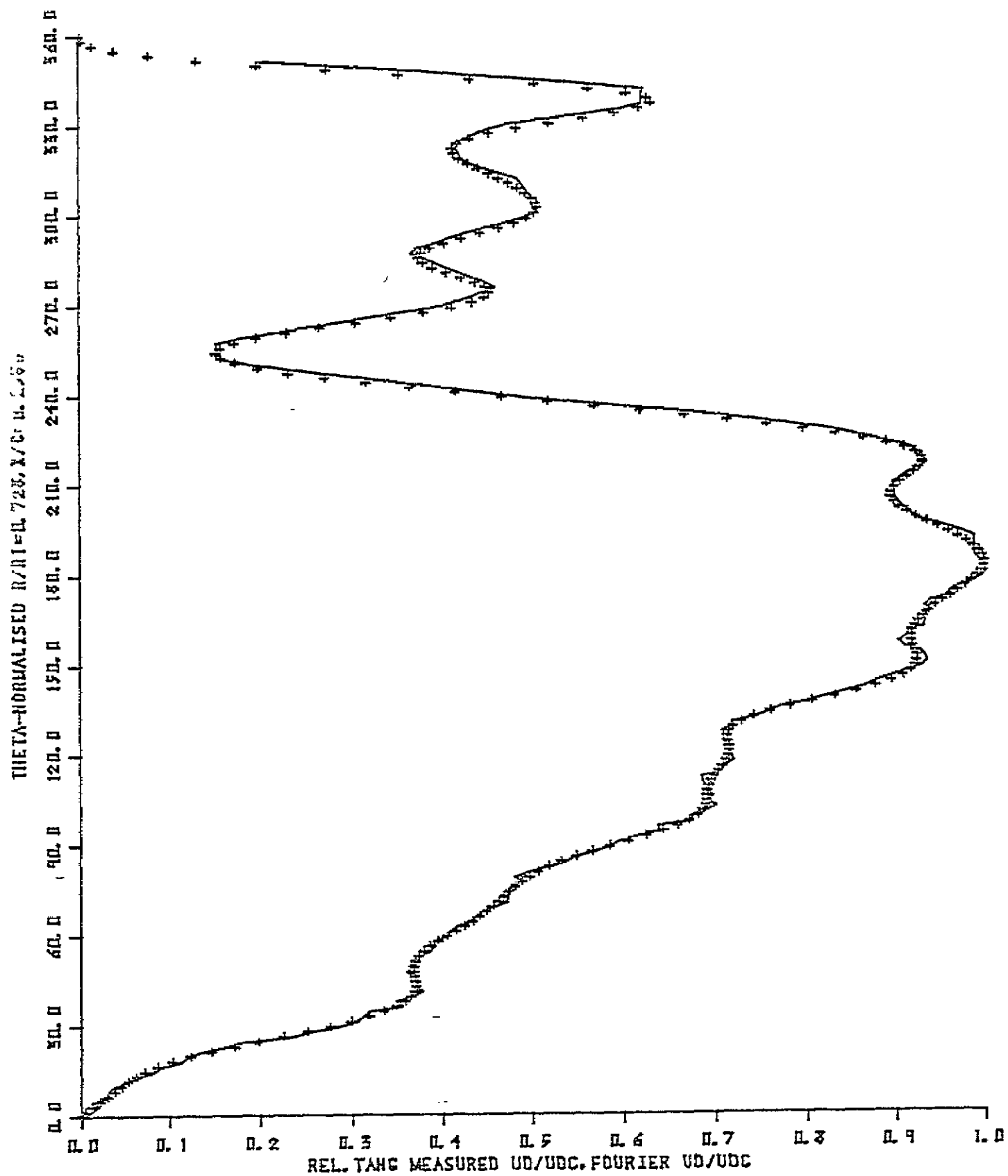


FIG. 59 COMPARISON OF FOURIER CURVE AND DATA

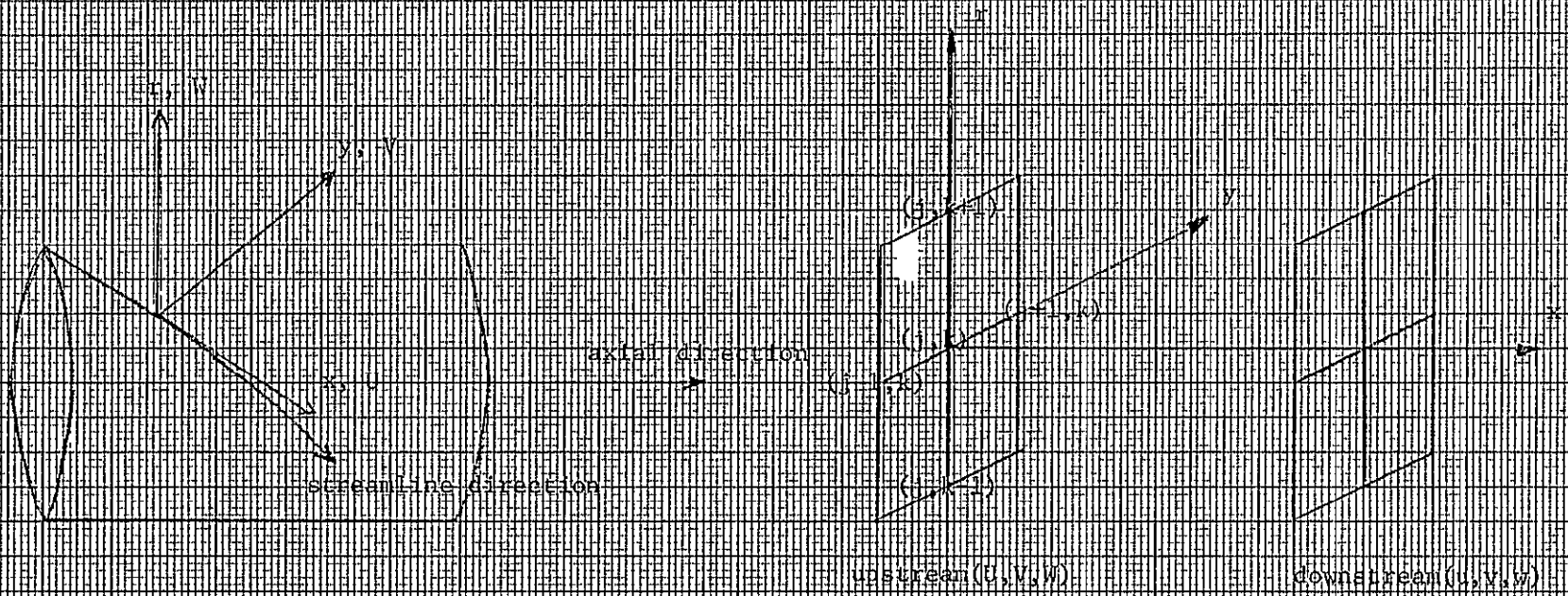


Fig. 60 Coordinate system for numerical analysis